Statistical Analysis of Antenna Array Systems with Perturbations in Phase, Gain and Element Positions

Mohammad Hossein Moghaddam, Sina Rezaei Aghdam, and Thomas Eriksson

Chalmers University of Technology, Gothenburg, Sweden
Outline

• Motivations behind statistical analysis of antenna array systems

• Perturbation modeling

• Perturbation analysis

• Simulation results

• Conclusion and future works
Motivations behind statistical analysis of antenna array systems

• System model for analyzing the effect of variabilities due to manufacturing processes in beamformer modules
  — variability of phase in the manufacturing process
  — variability of gain in the manufacturing processes
  — variability of element positions in the manufacturing processes
• What will happen to beam pattern, array gain, and sidelobe levels in presence of these variabilities?
• How can we determine maximum allowable variations for a given performance penalty?
Perturbation modeling, a bit of history
Perturbation modeling

- Phase shifter modeling for analyzing the electromagnetic beam of a linear array with $N$ elements

$$B(\theta, \psi) = B(k) = w^H v(k) = \sum_{i=0}^{N-1} g_i e^{j(\phi_i)} e^{-jkp_i}$$

- Variability modeling for phase, gain and element positions*

$$B(k) = \sum_{i=0}^{N-1} g_i (1 + \Delta g_i) e^{j(\phi_i + \Delta \phi_i) - jkp_i}$$

$$k = \frac{2\pi}{\lambda} \begin{bmatrix} \sin(\theta)\cos(\psi) & \sin(\theta)\sin(\psi) & \cos(\theta) \end{bmatrix}$$

- Where all perturbations are considered as uncorrelated zero-mean Gaussian random variables

Perturbation analysis

- Variability in manufacturing processes of phase $e^{j(\phi_i + \Delta \phi_i)}$

beam pattern realizations for $\sigma_g^2 = 0$, $\sigma_\phi^2 = 0.001$, and $\sigma_\lambda^2 = 0$

beam pattern realizations for $\sigma_g^2 = 0$, $\sigma_\phi^2 = 0.01$, and $\sigma_\lambda^2 = 0$
Perturbation analysis

• Variability in manufacturing processes of gain $g_i(1 + \Delta g_i)$
Perturbation analysis

- Variability in manufacturing processes of position of elements

\[ p_i = p_i^c + [0 \ 0 \ \Delta p_i]^T \]
Perturbation analysis

- Variations of beam power
  \[ \Lambda = \mathbb{E}[|B(k)|^4] - (\mathbb{E}[|B(k)|^2])^2 \]

- Mean of beam power
  \[
  \mathbb{E}[|B(k)|^2] = |B^c(k)|^2 e^{-(\sigma^2_{\phi} + \sigma^2_{\chi})} + ((1 + \sigma^2_{g}) - e^{-(\sigma^2_{\phi} + \sigma^2_{\chi})}) \sum_{i=0}^{N-1} g_i^2
  \]

- And then we need to calculate
  \[
  \mathbb{E}[|B(k)|^4] = \mathbb{E}[B(k)^H B(k) B(k)^H B(k)]
  \]
Perturbation analysis

- Variations of beam power

\[ \Lambda = \mathbb{E}[|B(k)|^4] - (\mathbb{E}[|B(k)|^2])^2 \]

- Mean of beam power

\[
E[|B(k)|^2] = |B^c(k)|^2 e^{-(\sigma_\phi^2 + \sigma_\lambda^2)} + ((1 + \sigma_g^2) - e^{-(\sigma_\phi^2 + \sigma_\lambda^2)}) \sum_{i=0}^{N-1} g_i^2
\]

- And then we need to calculate

\[
E[|B(k)|^4] = E[B(k)^H B(k) B(k)^H B(k)]
= \sum_{i=0}^{N-1} \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \sum_{q=0}^{N-1} E\left[ g_i (1 + \Delta g_i) g_l (1 + \Delta g_l) g_m (1 + \Delta g_m) g_q (1 + \Delta g_q) \right]
\]

\[
e^{j(\phi_i + \Delta \phi_i - \phi_l - \Delta \phi_l + \phi_q + \Delta \phi_m - \phi_m - \Delta \phi_q)} e^{-j\mathbf{k} (\mathbf{p}_i - \mathbf{p}_l + \mathbf{p}_m - \mathbf{p}_q)}.
\]
Perturbation analysis

• Lemma 1*: expected value of multiplication of four jointly Gaussian random variables

\[ E[\Delta g_i \Delta g_l \Delta g_m \Delta g_q] = E[\Delta g_i \Delta g_l] E[\Delta g_m \Delta g_q] + E[\Delta g_i \Delta g_m] E[\Delta g_l \Delta g_q] + E[\Delta g_i \Delta g_q] E[\Delta g_l \Delta g_m], \]

• Lemma 2*: expected value of product of normal exponential random variables

\[ E\left[ \prod_{i=1}^{K} e^{a_i z_i} \right] = E\left[ e^{a^T z} \right] = e^{a^T m + 0.5 a^T \Sigma a}, \]

• Where \( m \) is the mean vector and \( \Sigma \) is the covariance matrix

Perturbation analysis

- Using Lemma 1 & 2

\[
\Lambda = \sum_{i=0}^{N-1} \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \sum_{q=0}^{N-1} \left( g_i g_l g_m g_q e^{j(\phi_i - \phi_l + \phi_m - \phi_q)} e^{-j k (p_i^c - p_l^c + p_m^c - p_q^c)} \right)
\]

\[
(1 + (\delta_{mq} + \delta_{im} + \delta_{iq} + \delta_{lm} + \delta_{lq} + \delta_{il}) \sigma_g^2 + (\delta_{il} \delta_{mq} + \delta_{im} \delta_{lq} + \delta_{iq} \delta_{lm}) \sigma_g^4)
\]

\[
\left( e^{\sigma^2_\phi \left( -2 + (\delta_{mq} - \delta_{im} + \delta_{iq} + \delta_{lm} - \delta_{lq} + \delta_{il}) \right)} \right) \left( e^{\sigma^2_\lambda \left( -2 + (\delta_{mq} - \delta_{im} + \delta_{iq} + \delta_{lm} - \delta_{lq} + \delta_{il}) \right)} \right)
\]

\[
- \left( |B^c(k)|^2 e^{-\sigma^2_\phi + \sigma^2_\lambda} + \left((1 + \sigma_g^2) - e^{-\sigma^2_\phi + \sigma^2_\lambda}\right) \sum_{i=0}^{N-1} g_i^2 \right)^2
\]
Simulation results

- Monte-Carlo simulations for 100 realizations
- One standard deviation = 0.0188
Simulation results

- Statistical bounds
- One standard deviation = 0.0200
- Maximum three standard deviations is considered (0.06)
Simulation results

- Side-lobe, main-lobe variation analysis
Simulation results

- Tolerance analysis
- For a certain amount of variation in manufacturing process, we can determine maximum allowable variations for a given performance penalty

$$\sigma^2_g = 0.001, \sigma^2_{\lambda} = 0.001$$
Conclusion & Future works

• We model the phase shifters with three parameters for manufacturing process variability analysis, and can predict the maximum variations in the beam pattern.

• We can make tolerance study based on statistical derivations we made. We can indicate that for a certain amount of loss in the main lobe or a certain amount of increase in the side lobes, how much each parameter is free to variate in the manufacturing process.

• Future works:
  — We can make similar study for frequency selective variations, such as beam squint
  — We can make similar statistical modeling and analysis for other components in the beamformer module and take into account other impairments (Timing jitter, PA nonlinearities, …)
  — We can apply intended variations in the input parameters of Chalmers Massive MIMO Testbed (MATE) and measure the resulting beam patterns in the an-echoic chamber to show and validate our results in different scenarios