**Rate Maximization for Partially Connected Hybrid Beamforming in Single-User MIMO Systems**

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**Abstract**
- Key idea: Development of rate maximizing hybrid beamforming (HBF) algorithms for partially connected RF architecture.
- System: Downlink single-user MIMO system with perfect CSI.
- Problem: Rate maximization with Tx power constraint.
- Solution: Equivalent weighted minimum mean square error (WMMSE) problem solved by using alternating optimization between receive combiner, digital precoder and analog beamformer.
- Numerical results: Partially connected HBF provides good balance between hardware complexity and system performance.

**Introduction**
- Massive MIMO technology can efficiently exploit the vast spectral resources available at millimeter waves.
- Digital beamforming is impractical for massive MIMO implementation due to demanding hardware requirements (one RF chain per antenna).
- Hybrid beamforming (HBF) is a promising approach to implement massive MIMO since it supports multi-stream transmission with affordable hardware complexity (low number of RF chains).
- We study partially connected HBF (with phase and amplitude control) against digital and analog beamforming.

**System model**
- Partly connected Hybrid BF at the BS side.
- Digital BF at the users side.
- $N_t = N_s \leq N_r$

**Problem Formulation**
- The received signal vector at the user: $y = \text{HAD}s + z = \sum_{j=1}^{N_t} \text{H}_{j}a_{j}d_{j}^H + z$
- Digital Precoder: $D = \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1N_s} \\ d_{21} & d_{22} & \cdots & d_{2N_s} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N_t1} & d_{N_t2} & \cdots & d_{N_tN_s} \end{pmatrix}$
- Analog beamformer: $A = \begin{pmatrix} a_{1} & 0 & \cdots & 0 \\ 0 & a_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{N_s} \end{pmatrix}$

**Solution**
- Rate maximization problem with Tx power constraint:

$$\max \sum_{j=1}^{N_t} R_i \quad \text{s.t.} \quad \text{tr}(\text{ADD}^H A^H) \leq P$$

**Proposed Algorithms**
- Full array based WMMSE algorithm:
  - Set iteration number $n = 0$.
  - Initialize $D^0$ and $A^0$.
  - Repeat
    - Solve (11) for $M^0$ while $D^{n-1}$ and $A^{n-1}$ are fixed.
    - Compute $M^0$ from (6), (8) gives $D^{n-1}$, $A^{n-1}$, and $M^0$.
    - Solve (12) for $D^n$ while $M^0$ and $A^{n-1}$ are fixed.
    - Solve (13) for $[a^n]$ while $M^n$ and $D^n$ are fixed.
  - Until convergence
- Subarray based WMMSE algorithm: Digital precoder is considered to be identity, $D = I$.

**Convergence Results**

**Rate vs. SNR**

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>Average Rate (b/s/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Full Array-Based Hybrid WMMSE</td>
</tr>
<tr>
<td>20</td>
<td>Subarray-Based Hybrid WMMSE</td>
</tr>
<tr>
<td>25</td>
<td>Analog Beamforming</td>
</tr>
</tbody>
</table>

**Conclusion**
- Partially connected HBF obtains a good compromise between achievable rate and hardware complexity in comparison to digital and analog beamforming.
- Performance of full and subarray-based WMMSE algorithms are comparable for $N_s \leq 4$ at medium/high SNRs.
- Rate maximizing results serve as upper bounds for lower complexity HBF algorithms.