1 Introduction

Segmentation is the partition of the image into regions which share common features. We propose a variational segmentation method that considers adaptive patches to characterize, in a fast way, the local structure of each homogeneous region of the image. To be able to give a sharp representative disc associated to each region we use the $L_1$-norm.

Goals

- Characterizing the image regions that have homogeneous texture regardless of the point of view.
- Obtaining the region representative disc.

3 Proposed Model

Let $u : \Omega \to \mathbb{R}^H$ be an image with a patch $p_i(x)$ associated to each pixel and $P_u = \{p_i(x), x \in \Omega\}$ the set of all patches. From now on, with an abuse of notation, $p_i(x)$ will denote the normalized discs. We propose to simplify $P_u$ by estimating a representative set of discs $\{p_{10}, \ldots, p_{1N}\}$. It is modelled with the energy:

$$E(p, \omega) = \sum_{i=1}^N \int_N \left| \nabla \chi_0(x) \right| dx + \lambda \sum_{i=1}^N \int \nabla D_i(p(x), p(x)) \chi_0(x) dx,$$

where $p = \sum p_i \chi_0, \Omega$, is a region.

Optimization.

1. Characteristic functions belong to a space which is not convex: change to fuzzy membership functions $\omega$:

   $$E(\omega) = \sum_{i=1}^N \int_N \left| \nabla \chi_0(x) \right| dx + \lambda \sum_{i=1}^N \int D_i^{-1}(p(x), p(x)) \omega_i(x) dx.$$

2. We introduce an auxiliary variable $v$ to decouple the optimization problem:

   $$E(P, \omega, v) = E(\omega) + \lambda E(v, p) + \frac{1}{2} \sum_{i=1}^N \int (\omega_i(x) - v_i(x))^2 dx.$$

3. As $E$ is convex w.r.t each variable, it is minimized by alternatively fixing two variables and minimize w.r.t the third one, and iterate until convergence:

   - **v-subproblem:** Dual Formulation $[3]$:
     $$\omega_i(x) = \nu_i(x) = \nu_i(x) + \delta \chi_0(p(x))$$
     $$\hat{\nu}_i^{-1}(x) = \hat{\nu}_i^{-1}(x) + \nabla D_i^{-1}(p(x), p(x))$$
     $$\nu_i(x) = \min \{\max \{\omega_i(x) - \lambda D_i^{-1}(p(x), p(x)), 0\}, 1\}$$

   - **p-subproblem:** the solution is given by a weighted median vector $[1]$: $p(x)$ is the representative disc associated to each pixel and $\hat{\nu}_i^{-1}(x)$ the characteristic function output:
     $$\sum_{i=1}^N \nu_i(x) p_i(x) = \sum_{i=1}^N \nu_i(x) p_i(x)$$

Postprocessing and Output

Fuzzy functions output:

$$\nu_i(x) = \begin{cases} 1, & \text{if } i = \arg \max \omega_i(x) \\ 0, & \text{else} \end{cases}$$

Characteristic functions output and associated discs:

$$E(\omega) = \sum p_i \nu_i \chi_0 \omega_i$$

4 Results

Input & patch Li et. al. [5] Ours Discs

Initialization. As the functional is not jointly-convex the final result has a high dependence on the initialization. We initialize the algorithm using FCM [2]:

$$\omega_{k+1}(x) = \frac{\sum_{i=1}^N D_i^{-1}(p_i(x), p(x))}{\sum_{i=1}^N (D_i^{-1}(p_i(x), p(x)))^2} \forall x, k$$

5 Conclusions

Our model considers:
- Similarity among discs in an $L_1$ data term.
- TV of fuzzy membership functions as relaxed length of the region boundaries.

We provide as output:
- Partition of the image into local homogeneous regions.
- Texture disc associated to each region.

6 Future work

- Use patch texture features to compare the patches.
- Fill-in the region with the texture from the representative patch.

7 References