Globally Optimal Beamforming for Rate Splitting Multiple Access

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Rate-Splitting Multiple Access (RSMA)

Spatial and power domains user multiplexing using linearly precoded rate splitting & SIC

Properties:
- Partially decode interference and partially treat interference as noise
- Bridge the extremes of NOMA and SDMA (more general and powerful)

Benefits:
- Special cases: SDMA, NOMA, OMA and multicasting
- Improved Spectral and Energy Efficiency
- Optimal DoF for perfect and imperfect CSIT
- Robust against arbitrary user deployments
- CSIT inaccuracy
- Network load
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More on RSMA

IEEE ComSoc Special Interest Group on RSMA:

https://sites.google.com/view/ieee-comsoc-wtc-sig-rsma

Upcoming tutorial:

- IEEE ICC 2021 Tutorial on Rate Splitting Multiple Access for Beyond 5G: Principles, Recent Advances, and Future Research Trends, Montreal, Canada
- Date: 14–18 June 2021
- Speakers: Prof. Bruno Clerckx, Dr. Yijie (Lina) Mao
- More info: https://icc2021.ieee-icc.org/program/tutorials#tut-03
System Model

- $M$ transmit antennas and $K$ single-antenna users: $\mathcal{K} = \{1, \ldots, K\}$
- Message Splitting: $W_k \xrightarrow{\text{split}} \{W_{c,k}, W_{p,k}\}$
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- Common stream: $\{W_{c,1}, \ldots, W_{c,K}\} \xrightarrow{\text{encode}} s_c$, Private streams: $W_{p,k} \xrightarrow{\text{encode}} s_k$
**System Model**

- $M$ transmit antennas and $K$ single-antenna users: $\mathcal{K} = \{1, \ldots, K\}$
- Message Splitting: $W_k \xrightarrow{\text{split}} \{W_{c,k}, W_{p,k}\}$
- Common stream: $\{W_{c,1}, \ldots, W_{c,K}\} \xrightarrow{\text{encode}} s_c$, Private streams: $W_{p,k} \xrightarrow{\text{encode}} s_k$
- Linear Precoding: $x = p_c s_c + p_1 s_1 + \ldots + p_K s_K$
- Average power constraint: $\|p_c\|^2 + \sum_{k \in \mathcal{K}} \|p_k\|^2 \leq P$

All users decode $s_c$ first, before decoding $s_k$ (for user $k$)

**Rate of user-$k$ has been split:**
rate of $s_k +$ part of the rate of $s_c$
Problem Statement

\[
\begin{align*}
\max_{\mathbf{p}_1, \ldots, \mathbf{p}_K, \mathbf{p}_c, \gamma_c, \gamma_p} & \quad \sum_{k \in \mathcal{K}} u_k \left( C_k + \log(1 + \gamma_{p,k}) \right) \\
\text{s.t.} & \quad \gamma_c \leq \min_k \left\{ \frac{|\mathbf{h}_k^H \mathbf{p}_c|^2}{\sum_{j \in \mathcal{K}} |\mathbf{h}_k^H \mathbf{p}_j|^2 + 1} \right\} \\
& \quad \gamma_{p,k} \leq \frac{|\mathbf{h}_k^H \mathbf{p}_k|^2}{\sum_{j \in \mathcal{K} \setminus k} |\mathbf{h}_k^H \mathbf{p}_j|^2 + 1} \\
& \quad \sum_{k' \in \mathcal{K}} C_{k'} \leq \log(1 + \gamma_c) \\
& \quad \forall k : C_k \geq \max \left\{ 0, \ R_k^{th} - \log(1 + \gamma_{p,k}) \right\} \\
& \quad \left\| \mathbf{p}_c \right\|^2 + \sum_{k \in \mathcal{K}} \left\| \mathbf{p}_k \right\|^2 \leq P
\end{align*}
\]
Problem Statement

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\max_{\mathbf{p}_1, \ldots, \mathbf{p}_K, \mathbf{p}_c, \mathbf{c}, \gamma_c, \gamma_p} & \quad \sum_{k \in \mathcal{K}} u_k \left( C_k + \log(1 + \gamma_{p,k}) \right) \\
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& \quad \gamma_{p,k} \leq \frac{|\mathbf{h}_k^H \mathbf{p}_k|^2}{\sum_{j \in \mathcal{K}\setminus k} |\mathbf{h}_k^H \mathbf{p}_j|^2 + 1} \\
& \quad \sum_{k' \in \mathcal{K}} C_{k'} \leq \log(1 + \gamma_c) \\
& \quad \forall k : C_k \geq \max \left\{ 0, R_{k}^{th} - \log(1 + \gamma_{p,k}) \right\} \\
& \quad \|\mathbf{p}_c\|^2 + \sum_{k \in \mathcal{K}} \|\mathbf{p}_k\|^2 \leq P
\end{align*}\]
Problem Statement

\begin{align*}
\max_{p_1, \ldots, p_K, p_c, c_1, c_2, \gamma_c, \gamma_p} & \quad \sum_{k \in \mathcal{K}} u_k \left( C_k + \log(1 + \gamma_{p,k}) \right) \\
\text{s.t.} & \quad \gamma_c \leq \min_k \left\{ \frac{|h_k^H p_c|^2}{\sum_{j \in \mathcal{K}} |h_k^H p_j|^2 + 1} \right\} \\
& \quad \gamma_{p,k} \leq \frac{|h_k^H p_k|^2}{\sum_{j \in \mathcal{K} \setminus \{k\}} |h_k^H p_j|^2 + 1} \\
& \quad \sum_{k' \in \mathcal{K}} C_{k'} \leq \log(1 + \gamma_c) \\
& \quad \forall k : C_k \geq \max \left\{ 0, R_k^{th} - \log(1 + \gamma_{p,k}) \right\} \\
& \quad \|p_c\|^2 + \sum_{k \in \mathcal{K}} \|p_k\|^2 \leq P \quad \text{Tx Power Constraint}
\end{align*}

\( \mu = 0, P_c = 1 \): Weighted Sum Rate

\( u_k = 1 \): Energy Efficiency
Problem Statement

\[
\max_{\mathbf{p}_1, \ldots, \mathbf{p}_K, \mathbf{p}_c, \mathbf{c}, \gamma_c, \gamma_p} \sum_{k \in \mathcal{K}} u_k \left( C_k + \log(1 + \gamma_{p,k}) \right) \\
\frac{\mu}{\mu} \left( \| \mathbf{p}_c \|^2 + \sum_{k \in \mathcal{K}} \| \mathbf{p}_k \|^2 \right) + P_c
\]

\[
\begin{align*}
\mu &= 0, P_c = 1: \text{Weighted Sum Rate} \\
u_k &= 1: \text{Energy Efficiency}
\end{align*}
\]

\[
\text{s.t.} \quad \gamma_c \leq \min_k \left\{ \frac{| \mathbf{h}_k^H \mathbf{p}_c |^2}{\sum_{j \in \mathcal{K}} | \mathbf{h}_k^H \mathbf{p}_j |^2 + 1} \right\}
\]

\[
\gamma_{p,k} \leq \frac{| \mathbf{h}_k^H \mathbf{p}_k |^2}{\sum_{j \in \mathcal{K} \setminus k} | \mathbf{h}_k^H \mathbf{p}_j |^2 + 1}
\]

\[
\sum_{k' \in \mathcal{K}} C_{k'} \leq \log(1 + \gamma_c)
\]

\[
\forall k: C_k \geq \max \left\{ 0, R_{k}^{th} - \log(1 + \gamma_{p,k}) \right\}
\]

\[
\| \mathbf{p}_c \|^2 + \sum_{k \in \mathcal{K}} \| \mathbf{p}_k \|^2 \leq P \quad \text{Tx Power Constraint}
\]
Problem Statement

\[
\begin{align*}
\max_{\mathbf{p}_1, \ldots, \mathbf{p}_K, \mathbf{p}_c, \mathbf{c}, \gamma_c, \gamma_p} & \quad \sum_{k \in \mathcal{K}} u_k \left( C_k + \log(1 + \gamma_{p,k}) \right) \\
& \quad \mu \left( \|\mathbf{p}_c\|^2 + \sum_{k \in \mathcal{K}} \|\mathbf{p}_k\|^2 \right) + P_c \\
\text{s.t.} & \quad \gamma_c \leq \min_k \left\{ \frac{|h_k^H \mathbf{p}_c|^2}{\sum_{j \in \mathcal{K}} |h_k^H \mathbf{p}_j|^2 + 1} \right\} & \text{Common Msg SINRs} \\
& \quad \gamma_{p,k} \leq \frac{|h_k^H \mathbf{p}_k|^2}{\sum_{j \in \mathcal{K}\setminus k} |h_k^H \mathbf{p}_j|^2 + 1} \\
& \quad \sum_{k' \in \mathcal{K}} C_{k'} \leq \log(1 + \gamma_c) & \text{Common Msg Rate Allocation} \\
& \quad \forall k : C_k \geq \max \left\{ 0, R_{th}^k - \log(1 + \gamma_{p,k}) \right\} & \text{Minimum Rate QoS \& Nonnegativity} \\
& \quad \|\mathbf{p}_c\|^2 + \sum_{k \in \mathcal{K}} \|\mathbf{p}_k\|^2 \leq P
\end{align*}
\]
Problem Statement

\[ \max_{p_1, \ldots, p_K, p_c, c, \gamma_c, \gamma_p} \sum_{k \in K} u_k \left( C_k + \log(1 + \gamma_{p,k}) \right) \]
\[ \frac{\mu \left( \|p_c\|^2 + \sum_{k \in K} \|p_k\|^2 \right)}{\mu \left( \|p_c\|^2 + \sum_{k \in K} \|p_k\|^2 \right) + P_c} \]

s.t. \[ \gamma_c \leq \min_k \left\{ \frac{|h_k^H p_c|^2}{\sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1} \right\} \]
\[ \gamma_{p,k} \leq \frac{|h_k^H p_k|^2}{\sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1} \]
\[ \sum_{k' \in K} C_{k'} \leq \log(1 + \gamma_c) \]
\[ \forall k : C_k \geq \max \left\{ 0, R_{th}^{k} - \log(1 + \gamma_{p,k}) \right\} \]
\[ \|p_c\|^2 + \sum_{k \in K} \|p_k\|^2 \leq P \]

Common Msg SINRs

Common Msg Rate Allocation

Minimum Rate QoS & Nonnegativity
Problem Statement

\[ \begin{align*}
\max_{p_1, \ldots, p_K, p_c, c, \gamma_c, \gamma_p} & \quad \frac{\sum_{k \in K} u_k (C_k + \log(1 + \gamma_{p,k}))}{\mu \left( \|p_c\|^2 + \sum_{k \in K} \|p_k\|^2 \right)} + P_c \\
\text{s.t.} & \quad \gamma_c \leq \min_k \left\{ \frac{\|h_k^H p_c\|^2}{\sum_{j \in K} \|h_k^H p_j\|^2 + 1} \right\} \\
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& \quad \|p_c\|^2 + \sum_{k \in K} \|p_k\|^2 \leq P
\end{align*} \]

Problem Types:
- Unicast Beamforming
- Multicast Beamforming

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Problem Statement

\[
\max_{p_1, \ldots, p_K, p_c, c, \gamma_c, \gamma_p} \frac{\sum_{k \in \mathcal{K}} u_k (C_k + \log(1 + \gamma_{p,k}))}{\mu \left( \|p_c\|^2 + \sum_{k \in \mathcal{K}} \|p_k\|^2 \right) + P_c}
\]

s.t.

\[
\gamma_c \leq \min_k \left\{ \frac{|h_k^H p_c|^2}{\sum_{j \in \mathcal{K}} |h_k^H p_j|^2 + 1} \right\}
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\gamma_{p,k} \leq \frac{|h_k^H p_k|^2}{\sum_{j \in \mathcal{K} \setminus k} |h_k^H p_j|^2 + 1}
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\frac{1}{\mu} \left( \|\mathbf{p}_c\|^2 + \sum_{k \in \mathcal{K}} \|\mathbf{p}_k\|^2 \right) + P_c
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\text{s.t.} \quad \gamma_c \leq \min_k \left\{ \frac{|h_k^H \mathbf{p}_c|^2}{\sum_{j \in \mathcal{K}} |h_k^H \mathbf{p}_j|^2 + 1} \right\}
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s.t. \[\gamma_c \leq \min_{k} \left\{ \frac{|h_k^H \ p_c|^2}{\sum_{j \in K} |h_k^H \ p_j|^2 + 1} \right\} \]

\[\gamma_{p,k} \leq \frac{|h_k^H \ p_k|^2}{\sum_{j \in K \setminus k} |h_k^H \ p_j|^2 + 1} \]

\[\sum_{k' \in K} C_{k'} \leq \log(1 + \gamma_c) \]

\[\forall k : C_k \geq \max\left\{ 0, \ R_{th}^k - \log(1 + \gamma_{p,k}) \right\} \]

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- Unicast Beamforming
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NP-hard:
- Sum Rate Power Allocation [1]
- Multicast Beamforming [2]

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Problem Types:
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NP-hard:
- Sum Rate Power Allocation [1]
- Multicast Beamforming [2]

Observation:
Optimal Beamformers are rotationally invariant [3]:
\[
p_k^* = p_k^* e^{j \phi_k}
\]

Global Solution

- Multiextremal problem
Global Solution

- Multiextremal problem
- **Goal:** Global maximum

![Diagram showing a nonconcave function with local and global maxima marked.](image-url)
• Multiextremal problem
• **Goal:** Global maximum
• Polynomial time methods: Find local maximum at most
• Branch-and-Bound type algorithm
Global Solution

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- Branch-and-Bound principle:
  - Partition feasible set systematically

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**Branch-and-Bound principle:**
- Partition feasible set systematically
- On each partition element: Compute upper and lower bound on feasible objective values
- Successively refine partition
- If convergence criteria met: Upper − Lower \(\to 0\) as size(partition elements) \(\to 0\)
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Iteration $k_2 > k_1$

$$f(x^{k_2})$$ $f(x^*)$ $\beta_{\text{max}}$$
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Iteration $k_3 > k_2$

$\beta_{\text{max}} - f(\bar{x}^k) \to 0$ as $k \to \infty$
Global Solution

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- **Branch-and-Bound principle**:
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  - Successively refine partition
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- **Iteration** \( k_1 \)

- \( f(\bar{x}^{k_1}) + \eta \)
- \( f(\bar{x}) \)
- \( \beta_{\text{max}} \)
Global Solution

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Iteration $k_3$

Overlap $\Rightarrow \bar{x}^k$ is $\eta$-optimal
Global Solution

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- **Goal:** Global maximum
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**Iteration** \( k_3 \)

\[
\begin{align*}
\hat{f}(x^{k_3}), & \quad f(x^*) \\
\beta_{\text{max}} & \\
\text{Overlap} \Rightarrow \bar{x}^k \text{ is } \eta\text{-optimal}
\end{align*}
\]

- **Finite** Procedure (i.e., an algorithm) only if convergence provably after \( < \infty \) iterations
Unicast Beamforming

• Challenging constraint:

\[ \gamma_{p,k} \leq \frac{|h_k^H p_k|^2}{\sum_{j \in \mathcal{K} \setminus k} |h_k^H p_j|^2 + 1} \]

\[ \iff |h_k^H p_k|^2 \geq \gamma_{p,k} \left( \sum_{j \in \mathcal{K} \setminus k} |h_k^H p_j|^2 + 1 \right) \]
Unicast Beamforming

• Challenging constraint:

\[
\gamma_{p,k} \leq \frac{|h_k^H p_k|^2}{\sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1}
\]

\[
\Leftrightarrow \quad |h_k^H p_k|^2 \geq \gamma_{p,k} \left( \sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1 \right)
\]

• Exploit rotational invariance: Fix \( h_k^H p_k \) as nonnegative real \([1]\)

  • New constraints: \( \Re \{h_k^H p_k\} \geq 0 \) and \( \Im \{h_k^H p_k\} = 0 \)

---


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\[ \gamma_{p,k} \leq \frac{|h_k^H p_k|^2}{\sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1} \]

\[ \Leftrightarrow |h_k^H p_k|^2 \geq \gamma_{p,k} \left( \sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1 \right) \]

• Exploit rotational invariance: Fix \( h_k^H p_k \) as nonnegative real [1]

  • New constraints: \( \Re\{h_k^H p_k\} \geq 0 \) and \( \Im\{h_k^H p_k\} = 0 \)

• Second order cone constraint: \( h_k^H p_k \geq \sqrt{\gamma_{p,k} \left( \sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1 \right)} \)

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  \[ \gamma_{p,k} \leq \frac{|h_k^H p_k|^2}{\sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1} \]

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  - New constraints: \( \Re\{h_k^H p_k\} \geq 0 \) and \( \Im\{h_k^H p_k\} = 0 \)

- Second order cone constraint: 
  \[ h_k^H p_k \geq \sqrt{\gamma_{p,k} \left( \sum_{j \in K \setminus k} |h_k^H p_j|^2 + 1 \right)} \]

- Convex for fixed \( \gamma_{p,k} \)

- Nonconvexity of unicast beamforming: only in SINR \( \gamma_{p,k} \)


Unicast Beamforming

• Challenging constraint:

$$\gamma_{p,k} \leq \frac{|h_k^H p_k|^2}{\sum_{j \in K \backslash k} |h_k^H p_j|^2 + 1}$$

$$\Leftrightarrow |h_k^H p_k|^2 \geq \gamma_{p,k} \left( \sum_{j \in K \backslash k} |h_k^H p_j|^2 + 1 \right)$$

• Exploit rotational invariance: Fix $h_k^H p_k$ as nonnegative real [1]
  • New constraints: $\Re\{h_k^H p_k\} \geq 0$ and $\Im\{h_k^H p_k\} = 0$

• Second order cone constraint: $h_k^H p_k \geq \sqrt[2]{\gamma_{p,k} \left( \sum_{j \in K \backslash k} |h_k^H p_j|^2 + 1 \right)}$

• Convex for fixed $\gamma_{p,k}$

• Nonconvexity of unicast beamforming: only in SINR $\gamma_{p,k}$

• Branch-and-Bound over $\gamma_{p,k}$ [2]

• Reduces nonconvex dimension from $2N$ to 1


Multicast Beamforming

- Challenging constraint:

\[
\gamma_c \leq \min_k \left\{ \frac{|h_k^H p_c|^2}{\sum_{j \in \mathcal{K}} |h_k^H p_j|^2 + 1} \right\}
\]

\[
\Leftrightarrow \quad \forall k : |h_k^H p_c|^2 \geq \gamma_c \left( \sum_{j \in \mathcal{K}} |h_k^H p_j|^2 + 1 \right)
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• Rotational invariance helps only with one expression (do it for \(k = 1\))
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- **Result:** BB procedure over \((\gamma_p, \gamma_c, [\angle(h_c^H p_k)]_{k>1})\) with SOC bounding problem

Successive Incumbent Transcending Scheme

- Complexity (= nonconvexity) is in feasible set → Convergence issues in BB procedures
Successive Incumbent Transcending Scheme

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State-of-the-Art (unicast beamforming):
- Direct implementation of SOC bounding problem
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  - Line search to obtain feasible point in every iteration
  - Adds several SOCP for each partition element
  - Increased computational complexity but finite convergence

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- Modified BB procedure: numerically stable & guaranteed finite convergence
- Details: Paper & ICASSP Tutorial T-3
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Conclusions

Contributions:

- Successive incumbent transcending BB algorithm for joint unicast & multicast BF
  - Numerically stable & fast convergence (compared to SoA)
  - Special cases: NOMA, OMA, multicast beamforming

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