On the Age of Information in Multi-Source Multi-Hop Wireless Status Update Networks

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The Importance of Timely Information

- Cars send status updates to other cars
- Cars want the newest state information with low delay
- Metric: Age (staleness) of the most recent update
System Model / Assumptions

- We consider a multi-source multi-hop status update network with \( N \) nodes modeled by an undirected graph \( G = (\mathcal{V}, \mathcal{E}) \).

- Each node keeps a table of all \( N \) time-varying processes in the network.

- Out of the \( N \) parameters in each node’s table, 1 is obtained directly from its local process and the remaining \( N - 1 \) parameters are obtained indirectly from other nodes by dissemination.

- Transmission is slotted, where during each time slot only one packet containing information on one process can be transmitted.

**Goal**: Derive fundamental bounds on the peak and average AoI and develop schedules that achieve near-optimal performance.
**Age**: Assume the most recent status update of the $H_i$ process received at node $j$ was timestamped at time $t'$. The age of status update $H_i^{(j)}$ at time $t \geq t'$ is defined as $\Delta_i^{(j)}(t) \triangleq t - t'$ for $j \neq i$. Since each node is assumed to have zero-delay access to the status of its local process, we have $\Delta_i^{(i)}(t) = 0$ for any $i \in V$ and $t$.

**Average Age**: The average age is defined as

$$\Delta_{avg} \triangleq \lim_{T \to \infty} \left[ \frac{1}{N^2 - N} \sum_{i,j \in V, i \neq j} \frac{1}{T - \bar{t}} \int_{\bar{t}}^{T} \Delta_i^{(j)}(t) \, dt \right]$$

for $\bar{t}$ sufficiently large such that all nodes have complete status update tables.
Aol Metrics

Peak Age: The peak age is defined as

$$\Delta_{peak} \triangleq \sup_{t \geq \bar{t}, i,j \in V, i \neq j} \Delta^{(j)}_i(t)$$

Schedule: We refer to a schedule as an ordered sequence of transmitting nodes and the corresponding update parameter that they disseminate in each time slot.
### Example Line Network

![Diagram of a line network with nodes 1, 2, and 3 connected in a line]

**Table: Example schedule for the 3-node line network.**

<table>
<thead>
<tr>
<th>time slot</th>
<th>transmitting node</th>
<th>disseminated update</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 0, 5, 10, \ldots$</td>
<td>1</td>
<td>$H_1^{(1)}$</td>
</tr>
<tr>
<td>$n = 1, 6, 11, \ldots$</td>
<td>2</td>
<td>$H_1^{(2)}$</td>
</tr>
<tr>
<td>$n = 2, 7, 12, \ldots$</td>
<td>2</td>
<td>$H_2^{(2)}$</td>
</tr>
<tr>
<td>$n = 3, 8, 13, \ldots$</td>
<td>3</td>
<td>$H_3^{(3)}$</td>
</tr>
<tr>
<td>$n = 4, 9, 14, \ldots$</td>
<td>2</td>
<td>$H_3^{(2)}$</td>
</tr>
</tbody>
</table>
Operation of a schedule for the 3-node line network
Key Graph Parameters

- Average distance $\ell_G \triangleq \frac{1}{N^2 - N} \sum_{i,j \in V, i \neq j} d(i, j)$
- Minimum connected dominating set (MCDS) $S$
- Connected domination number $\gamma_c$
- Pseudo-leaf vertex $u$, $u \in \mathcal{L} \triangleq V - \{S_1 \cup S_2 \cup \ldots \cup S_M\}$

![Graph diagram with nodes and sets labeled $S_1 = \{2,3\}$ and $S_2 = \{2,4\}$]
Fundamental Bounds on AoI of $T^*$-periodic Schedules

**Lemma 1.** [Lower bound on the schedule length to refresh all tables] To update the status of all parameters throughout the network, at least $T \geq T^* \triangleq \gamma_c N + |\mathcal{L}|$ status update packets need to be disseminated.

**Theorem 1.** [Lower bound on peak age] The peak age of information for any $T^*$-periodic schedule is lower-bounded by

$$\Delta_{peak} \geq \Delta_{peak}^* \triangleq \begin{cases} 
\gamma_c(N + 1) & |\mathcal{L}| = 0 \\
\gamma_c(N + 1) + |\mathcal{L}| + 1 & |\mathcal{L}| \geq 1
\end{cases}.$$

**Theorem 2.** [Lower bound on average age] The average age of information for any $T^*$-periodic schedule is lower-bounded by

$$\Delta_{avg} \geq \Delta_{avg}^* \triangleq \frac{T^*}{2} + \ell_G.$$
Near-Optimal Schedule

Algorithm 1: Schedule design to disseminate status updates throughout the network

Step I: initialize time, $t \leftarrow -1$.

Step II: for node $i = 1 : N$ do

\* if $\exists$ MCDS $\bar{S}$ s.t. $i \in \bar{S}$ then
\* \hspace{1em} $S \leftarrow \bar{S}$.
\* else
\* \hspace{1em} $S \leftarrow \bar{S} \cup \{i\}$, for any MCDS $\bar{S} \subset V$.
\* end

\* $S_{\text{sorted}} = \text{Depth-First Search}(G[S], i)$

\* for $k = 1 : |S_{\text{sorted}}|$ do

\* \hspace{1em} $j = S_{\text{sorted}}(k)$,

\* node $j$ transmits $H_i^{(j)}(t^+)$,

\* $t \leftarrow t + 1$.

end

end

Step III: repeat from Step II.
Main Concept of the Near-Optimal Schedule Design

**Step 1:** Node 1 disseminates its current local information on the $H_1$ process

**Step 2:** Disseminate this information on the $H_1$ process to all nodes in the network through a MCDS

**Step 3:** Repeat for the local processes of nodes 2, 3, 4, ..., $N$ from Step 1

**Step 4:** Repeat from Step 1
Theorem 3. [Achievable peak age of the near-optimal schedule] The schedule generated by Algorithm 1 achieves $\Delta_{peak} = \Delta_{peak}^*$. 

Theorem 4. [Achievable average age of the near-optimal schedule] The average age of the schedule generated by Algorithm 1 is bounded by

$$\Delta_{avg} \leq \frac{T^*}{2} + \gamma_c + \frac{|L|}{N} \leq \Delta_{avg}^* + (N - 2)$$
Bounds and achievable AoI for every connected graph with $N \leq 9$

- $\Delta_{peak,ach}$, Algorithm 1
- $\Delta_{peak}$, Theorem 1
- $\Delta_{avg,ub}$, Theorem 4
- $\Delta_{avg,ach}$, Algorithm 1
- $\Delta_{avg}$, Theorem 2

achieved peak age exactly matches lower bound on peak age on all graphs

upper bound on average age for the schedules

achieved average age close to lower bound on average age on all graphs

sorted topology index

$\times 10^5$
Aol for networks generated based on Erdos-Renyi model

\[ \Delta \text{ peak} \]

\[ \Delta \text{ avg} \]

\[ N = 15 \]
\[ p = 0.30 \]
$\frac{\Delta_{\text{avg, ach}}}{\Delta^*_\text{avg}}$ for the Erdos-Renyi-based networks
Future Work

- Generalize the multi-source multi-hop model to consider arbitrary service time distributions
- Derive general bounds that completely characterize the AoI for any network topology and schedule
- Develop near-optimal schedule construction that is shown to analytically achieve the peak age bound and be within constant gap of average bound