On the Indistinguishability of Compressed Encryption With Partial Unitary Sensing Matrices

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CS-based Cryptosystems

• **Security for IoT and M2M**
  – Security issues are major challenges for the Internet-of-Things (IoT) and M2M communications.
  – Security techniques with **low latency, low power consumption**, and **low complexity** are required.

• **Compressed Sensing (CS) based Cryptosystems**
  – **Simultaneous sensing** and encryption
  – Efficient encryption/decryption
  – Reliability and security
  – Low complexity and low power consumption
CS-based Cryptosystems

• **History**
  - **Hint [Candes&Tao’06]**
    : CS measurement samples are *weakly encrypted*.
  - **Kick-off [Rachlin&Baron’08]**
    : CS-based cryptosystems cannot be *perfectly secure*, but can be computationally secure.
  - **Kick-off [Orsdemir et al.’08]**
    : Demonstrated that CS-based cryptosystems can be computationally secure.
  - **Gaussian one-time sensing (G-OTS) cryptosystem [Bianchi et al.’14]**
    : perfectly secure, as long as each plaintext has constant energy
  - **Random Bernoulli based cryptosystem [Cambareri et al.’15]**
    : CS-based cryptosystem for multiclass encryption
  
  – Many other research works for practical applications
    : smart grids, image encryption, wireless communications, etc.
CS-based Cryptosystems

- Symmetric-key CS-based Cryptosystems
CS-based Cryptosystems

- **CS Encryption/Decryption**

  Each plaintext is sparse with respect to an arbitrary basis.

  CS recovery algorithms are applied for CS decryption.

Key: $M \times N$ matrix
Plaintext: $N \times 1$ vector
Ciphertext: $M \times 1$ vector
CS-based Cryptosystems

- **Gaussian One-Time Sensing (G-OTS) Cryptosystem**

  - **One-time sensing**: a random Gaussian matrix is used only once, and renewed for each encryption.

  $\Phi$: random Gaussian matrix
CS-based Cryptosystems

- **Gaussian One-Time Sensing (G-OTS) Cryptosystem**
  - **Pros**
    - The G-OTS cryptosystem reveals only the energy of the plaintext.
    - Thus, it is *perfectly secure*, as long as each plaintext has constant energy.
  - **Cons**
    - Each CS encryption/decryption requires *high complexity* and *processing time* by matrix-vector multiplication with Gaussian distributed elements.
    - $M \times N$ Gaussian distributed elements are required for each encryption.

The motivation of this work is to overcome the practical concerns.
Proposed CS-based Cryptosystems

- Proposed CS encryption

\[ \text{Ciphertext} = \text{Plaintext} \times \text{Key} \]

Partial unitary matrix: \textit{public}

Unitary matrix: \textit{public}

Bipolar keystream: \textit{secret}
Proposed CS-based Cryptosystems

• Mathematical Formulation

\[
\Phi = \frac{1}{\sqrt{M}} R_{\Omega} U = \frac{1}{\sqrt{MN}} R_{\Omega} U_1 \text{diag}(s) U_2
\]

- \( U_1 = H \): Each entry of \( U_1 \) should have unit magnitude.

\[
H(k, t) = \begin{cases} 
1, & \text{if } k = 0 \text{ or } t = 0, \\
(-1)^{d_k t - 2}, & \text{otherwise}
\end{cases}
\]

- \( U_2 \): Unitary matrix
- \( s \): secret bipolar keystream
  - LFSR-based keystream
  - Example: Self-shrinking generator (SSG)

\( d \) is a binary \( m \)-sequence.

The secret keystream bits can be generated fast and efficiently.
Proposed CS-based Cryptosystems

• **Practical Benefits**
  – **Efficient keystream usage**
    • G-OTS cryptosystem: $M \times N$ real-valued elements required for each encryption
    • Proposed cryptosystem: $N$ keystream bits required for each encryption
  – **Fast and efficient keystream generation**: The original keystream can be efficiently generated by an LFSR-based keystream generator.
  – **Fast and efficient CS encryption/decryption**: By employing unitary matrices, matrix-vector multiplications for CS processes can be efficiently implemented.

• **Reliability**
  – **Stable and robust CS decryption**: A plaintext with at most $K$ nonzero entries can be decrypted with bounded errors by a legitimate recipient, as long as
    $$M = \mathcal{O}(K \log^5 N)$$
Security Analysis

• **Indistinguishability**
  – If a cryptosystem has the indistinguishability, no eavesdropper can learn any partial information about the plaintext from a given ciphertext.
  – The indistinguishability formalizes the notion of *computational security* of a cryptosystem.

  – The indistinguishability is measured by the success probability of an adversary in the *indistinguishability experiment*.
Indistinguishability Experiment (for a CS-based cryptosystem)

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>An adversary creates a pair of plaintexts $x_1$ and $x_2$ of the same length, and submits them to a CS-based cryptosystem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2:</td>
<td>The CS-based cryptosystem encrypts a plaintext $x_h$ by randomly selecting $h \in {1, 2}$, and gives a noisy ciphertext $r = \Phi x_h + n$ back to the adversary.</td>
</tr>
<tr>
<td>Step 3:</td>
<td>Given the ciphertext $r$, the adversary carries out a polynomial time test $\mathcal{D}: r \rightarrow h' \in {1, 2}$, to figure out the corresponding plaintext.</td>
</tr>
</tbody>
</table>

**Decision:** The adversary passes the experiment if $h' = h$, or fails otherwise.

- If no adversary passes the indistinguishability experiment in polynomial time with probability significantly better than that of a random guess, the cryptosystem is said to have the indistinguishability.
Security Analysis

• Total Variation (TV) Distance

\[ d_{TV}(\mu, \nu) = \sup_{A \subset \Omega} |\mu(A) - \nu(A)| \]

• \( \mu, \nu \): probability measures on \( \Omega \)
  – The success probability of an adversary in the indistinguishability experiment

\[ p_d \leq \frac{1}{2} + \frac{d_{TV}(p_1, p_2)}{2} \]

• \( p_1 = \Pr(r|x_1) \) and \( p_2 = \Pr(r|x_2) \)
  – The TV distance can be a statistical measure for indistinguishability.
Security Analysis

• Hellinger Distance

\[ d_H(\mu, \nu) = \left[ \frac{1}{2} \int_{\Omega} \left( \sqrt{f(x)} - \sqrt{g(x)} \right)^2 \, dx \right]^{\frac{1}{2}} \]

• \( f, g \): densities of probability measures \( \mu, \nu \) on \( \Omega \)

– For multivariate normal with zero mean,

\[ d_H^2(p_1, p_2) = 1 - \frac{\det(C_1)\frac{1}{4} \det(C_2)^{\frac{1}{4}}}{\det \left( \frac{C_1 + C_2}{2} \right)^{\frac{1}{2}}} \]

• \( C_1 \) and \( C_2 \): Covariance matrices of \( r \) conditioned on \( x_1 \) and \( x_2 \)
Security Analysis

• TV and Hellinger distances

\[
d_H^2(p_1, p_2) \leq d_{TV}(p_1, p_2) \leq d_H(p_1, p_2)\sqrt{2 - d_H^2(p_1, p_2)}
\]

**Theorem:** In the proposed CS-based cryptosystem, if each plaintext \( \mathbf{x} \) has at most \( K \) nonzero elements with constant energy \( \mathcal{E}_x \), then

\[
d_H(p_1, p_2) \leq \sqrt{1 - \left( \frac{2\mathcal{E}_x \mu^2(\mathbf{U}_2) \cdot \text{PNR} + 1}{K\mu^2(\mathbf{U}_2) \cdot \text{PNR} + 2} \right)^{\frac{M}{4}}}
\]

where \( \text{PNR} = \frac{\mathcal{E}_x}{M\sigma^2} \) and \( \mu(\mathbf{U}_2) \) is the maximum magnitude of the entries of \( \mathbf{U}_2 \).
For a legitimate recipient, the proposed CS-based cryptosystem is as reliable as the random Gaussian sensing.

- $N = 1024$
- $M = 48$
- $K = 4$
Numerical Results

- **Success probabilities**

For a given $M$, the adversary's success probability approaches that of a random guess as $N$ increases.

- $\text{PNR} = 25 \text{ dB}$
- $M = 48$
- $K = \left\lceil \frac{8.5M}{\log_2 N} \right\rceil$
Numerical Results

• Success probabilities

For a given $K$, the adversary’s success probability approaches that of a random guess as $N$ increases.

- $PNR = 25$ dB
- $K = 4$
- $M = \left[0.12K\log_2 N\right]$
Conclusions

• **Proposed CS-based cryptosystem**
  – CS-based cryptosystem with partial unitary matrices embedding a secret bipolar keystream
    – Theoretically guarantees **reliable decryption** for a legitimate recipient.
    – Demonstrates the potential of **computational security** against an eavesdropper, if the keystream is sufficiently long with low compression and sparsity ratios.

– Practical benefits
  • **Efficient usage of cryptographic primitives** by embedding a short keystream.
  • **Fast and efficient keystream generation** by LFSR-based keystream generators.
  • **Fast and efficient CS encryption/decryption** by employing unitary matrices.
Thank You!