POTENTIAL GAMES FOR DISTRIBUTED PARAMETER ESTIMATION IN NETWORKS WITH AMBIGUOUS MEASUREMENTS

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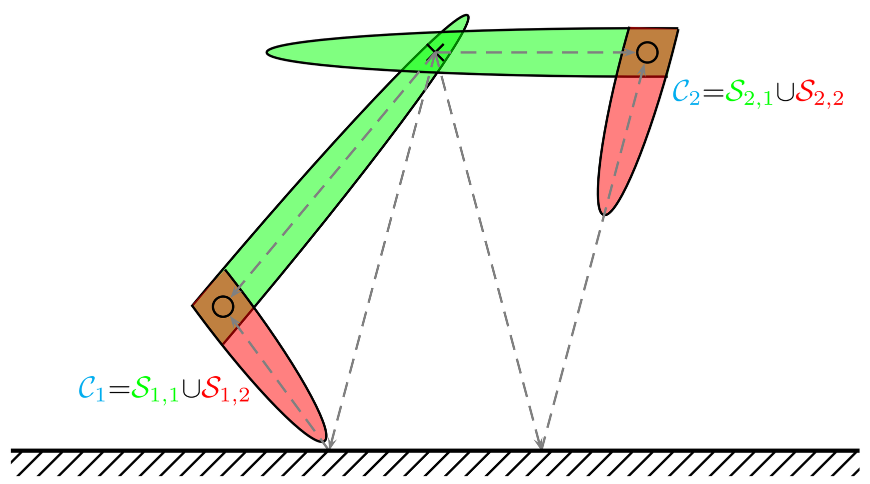
1. AT A GLANCE

- Distributed estimation of a parameter vector in a network of sensor nodes with ambiguous measurements is considered
- Non-convex constraint sets may be required at the nodes, in order to accurately model the local ambiguities
- The non-convexity is treated by expressing the involved non-convex sets as unions of convex sets, such that, for each node, only one such convex set is actually relevant
- The problem of selecting the relevant sets is modelled as a non-cooperative game, a potential function is derived, and an algorithm is proposed.

2. MOTIVATING EXAMPLE AND MODELING

- Consider a scenario in which two (or more) nodes utilize Angle of Arrival (AoA) measurements to localize a source, in an environment where reflections are present
- Some nodes compute multiple AoAs, however, only one is relevant to the source of interest



- Each node n adopts a set-theoretic approach, by considering that the unknown parameter vector $\theta \in \mathcal{C}_n$ where $\mathcal{C}_n$ is some proper, possibly non-convex, constraints set
- In this work we model such sets as

$$\mathcal{C}_n = \bigcup_{k=1}^{k_n} \mathcal{S}_{n,k},$$

where $\mathcal{S}_{n,k}$ denote convex sets and k_n is the number of such sets at node n, used to construct the non-convex set $\mathcal{C}_n$

3. PROBLEM FORMULATION

Consensus Problem P:

$$\text{Find } \theta \in \mathcal{C} = \bigcap_{n=1}^{N} \mathcal{C}_n ,$$

$$\text{where } \mathcal{C}_n = \bigcup_{k=1}^{k_n} \mathcal{S}_{n,k} ,$$

are non-convex sets, expressed as unions of the convex sets $\mathcal{S}_{n,k}$.

It constitutes a **particular form** of a non-convex feasibility problem.

4. PROBLEM DECOMPOSITION

Assumption A1: The intersection $\mathcal{C}$ is non-empty. Furthermore, there exists exactly one set $\mathcal{S}_{n,l_n}$ for each node n with

$$\mathcal{S}_{n,l_n} \bigcap \mathcal{C} \neq \emptyset$$

In other words, for each agent, there exists exactly one convex set, say $\mathcal{S}_{n,l_n}$ (selected among all $\mathcal{S}_{n,k}$ sets), whose intersection with all such sets of the other nodes is non-empty.

When Assumption A1 holds, the considered problem is equivalent to solving the following two sub-problems:

- **Sub-problem P1: Identify the sets $\mathcal{S}_{n,l_n}, n \in \mathcal{N}$, and**
- **Sub-problem P2: Compute $\theta \in \mathcal{S}_{n,l_n}$**

Sub-problem P2 has been extensively studied in literature, and can be solved by using the projections onto convex sets (POCS) approach[a]
The focus here is on sub-problem P1

[a] L.G. Gubin, B.T. Polyak, and E.V. Raik, "The method of projections for finding the common point of convex sets," USSR Computational Mathematics and Mathematical Physics, vol. 7, no. 6, pp. 1-24, 1967

5. A NON-COOPERATIVE POTENTIAL GAME FOR SUB-PROBLEM P1

Consider a non-cooperative game in strategic form:

- The set of players is the set of nodes $\mathcal{N}$. Each player has an action set

$$\mathcal{A}_n = \{\mathcal{S}_{n,1}, \mathcal{S}_{n,2}, \ldots, \mathcal{S}_{n,k_n}\}$$

- A strategy $\alpha_n \in \mathcal{A}_n$ for node/player n is the selection of one of its convex sets
- A strategy profile α is a selection of strategies, one for each player. Also, $\alpha \in \mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \ldots \mathcal{A}_N$ and $\alpha = (\alpha_n, \alpha_{-n})$

Utility function at node/player n with neighbourhood $\mathcal{N}_n$

$$u_n(\alpha) = \sum_{k \in \mathcal{N}_n} I(\alpha_n, \alpha_k) ,$$

where $I(\mathcal{S}_a, \mathcal{S}_b)$ is an indicator function defined as

$$I(\mathcal{S}_a, \mathcal{S}_b) = \begin{cases} 1, & \text{if } \mathcal{S}_a \bigcap \mathcal{S}_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases} .$$

- It counts the number of neighbours that have selected a set with non-empty intersection with the set selected by node n.

Following the work in[a], it can be proven that the function $\phi : \mathcal{A} \to \mathbb{R}$ defined as

$$\phi(\alpha) = \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{N}_n} \frac{I(\alpha_n, \alpha_k)}{2} ,$$

is a so-called *exact potential function*.[b]
An approach known as *Spatial Adaptive Play* (SAP)[c] can be used. According to this method, the nodes employ probabilities for their sets (strategies), which are updated from time $t-1$ to time t according to the rule

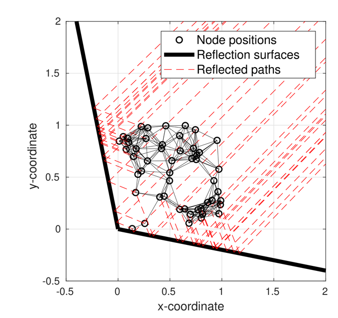
$$p_n(t, \alpha_n) = \frac{\exp\left(\beta u_n(\alpha_n, \alpha_{-n}(t-1))\right)}{\sum_{\alpha'_n \in \mathcal{A}_n} \exp\left(\beta u_n(\alpha'_n, \alpha_{-n}(t-1))\right)} ,$$

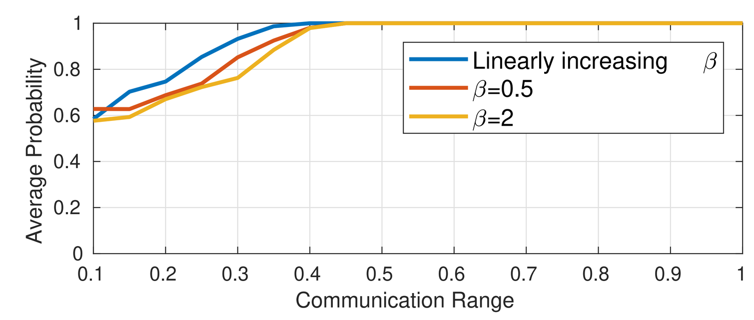
where $\beta \geq 0$ is the so-called *exploration parameter*, that controls how likely the players are to select a suboptimal strategy.
In a potential game where all players utilize SAP, the stationary distribution gives maximum probabilities to the strategies that jointly maximize the potential function[c].

[a] J. R Marden, G. Arslan, and J. S Shamma, "Connections between cooperative control and potential games illustrated on the consensus problem," in ECC 2007. IEEE, 2007, pp. 4604-4611
[b] D. Monderer and L. S. Shapley, "Potential games," Games and economic behavior, vol. 14, no. 1, pp. 124-143, 1996.
[c] H. P. Young, "Individual strategy and social structure",Princeton University Press, 1998

6. NUMERICAL RESULTS

- $N = 200$ nodes, uniformly deployed in the unit square (40 different realizations). A source is placed at (35,35). The signal is received directly and via two reflections
- Various communication ranges were tested, only realizations that resulted to connected graphs were considered. Nodes perform 6000 strategy changes.
- The average (across nodes and runs) probability for selecting the correct sets is given





We can see that, in all cases, the probability reaches the value 1, when the communication range is high enough, i.e., when the communication graph becomes more strongly connected.