A PARTIALLY COLLAPSED GIBBS SAMPLER FOR UNSUPERVISED NONNEGATIVE SPARSE SIGNAL RESTORATION

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Abstract

We introduce a new strategy, based on the Bernoulli-Generalized-Hyperbolic prior, to reconcile nonnegativity constraint with efficient sampling methods using partially collapsed Gibbs sampling (Marginalisation) for unsupervised nonnegative sparse signal restoration.

1. Nonnegative Sparse Signal Restoration

Problem statement

Goal: Find the sparse nonnegative vector \( \mathbf{x} \)

\[
J(\mathbf{x}) = \| \mathbf{y} - \mathbf{Hx} \|^2 \quad \text{s.t.} \quad \| \mathbf{x} \| \leq S, \quad x_k \geq 0 \quad \forall k
\]

→ Stochastic sampling: probabilistic hierarchical models.

Unsupervised case, and \( \mathbf{H} \) is highly correlated.

2. Available method: BTG Sampler

Bernoulli-Truncated-Gaussian prior \([1]\)

Let \( q \) binary variables such that \( \sum q_k = \| \mathbf{x} \|_0 \).

\[
\begin{align*}
q_k \in \{0, 1\} \\
\Pr(q_k = 1) = \xi
\end{align*}
\]

\[
\begin{align*}
x_k | q_k = 1 &\sim N(0, \sigma^2) \\
x_k | q_k = 0 &\sim \delta(x_k)
\end{align*}
\]

where \( N^+ \) is the truncated Gaussian.

\( \beta \) controls the skewness of the distribution (i.e., \( P(x_k \leq 0 | q_k = 1) \)).

3. Contribution: BGH Sampler

Bernoulli-Generalized-Hyperbolic prior (BGH)

\[
\begin{align*}
q_k \in \{0, 1\} \\
\Pr(q_k = 1) = \xi \\
x_k | q_k = 1 &\sim GH(\nu, \beta) \\
x_k | q_k = 0 &\sim \delta(x_k)
\end{align*}
\]

where \( \nu_{\beta} = \min_{\nu, \beta} TV(GH(\nu, \beta), N^+(0, 1)) \)

\( p(\mathbf{x}) \propto \exp \left( -\frac{1}{2\sigma^2} \| \mathbf{y} - \mathbf{Hx} \|^2 \right) p(\mathbf{q}) P(\mathbf{q} | \xi) \)

\( p(\mathbf{q}) = \frac{1}{2^S} \binom{S}{\sum q_k} \)

\( p(x_k | q_k = 1) \propto \exp \left( -\frac{1}{2\sigma^2} \| \mathbf{y} - \mathbf{Hx} \|^2 \right) \frac{\exp \left( -\frac{1}{2\beta^2} \| \mathbf{x} \|_2^2 \right)}{\int \exp \left( -\frac{1}{2\beta^2} \| \mathbf{x} \|_2^2 \right) d\mathbf{x}} \)

\( \mathbf{x} \) is efficiently marginalizable from \( p(x_k, w, q_k | \mathbf{y}, \theta) \).

for each \( k \)

Sample \( q_k | q_{-k}, x_{-k}, \theta, \mathbf{y} \) (Bernoulli)
Sample \( x_k | q_k, x_{-k}, \theta, \mathbf{y} \) (Truncated-Gaussian)

BGH-PCGS

for each \( k \)

Sample \( q_k, w_k | q_{-k}, w_{-k}, \theta, \mathbf{y} \) (from \( p(q_k, w_k | \mathbf{y}, \theta) \))
Sample \( x_k | q_k, w_k, \theta, \mathbf{y} \) (Gaussian, of size \( L = \| \mathbf{z} \|_0 \))
Sample \( \theta | q_k, x_k, w_k, \mathbf{y} \)

Efficient sampling using PCGS \([3]\)

4. Experiment

4.1 Sparse Decomposition

- \( \mathbf{x}^* \) BTG sequence.
- Noise level (SNR = 12 dB).
- Unsupervised scenario.
- For the BGH: \( \beta = 150 \).

4.2 Convergence Monitoring \([4]\)

MPSRF using \( r = 10 \) independent Markov chains.

Results

\( \text{MPSRF} \) w.r.t time

\( \text{Computing Time (s)} \)

Nonnegative restoration + Efficient sampling using PCGS.

5. Future Work

Approximate other models for nonnegativity:
- Bernoulli-Exponential.
- Exact decomposition: unconstrained case
- Bernoulli-Laplace, Bernoulli-Cauchy.
- Automatic tuning of parameter \( \beta \).

References