Matching Pursuit Based on Kernel Non-second Order Minimization

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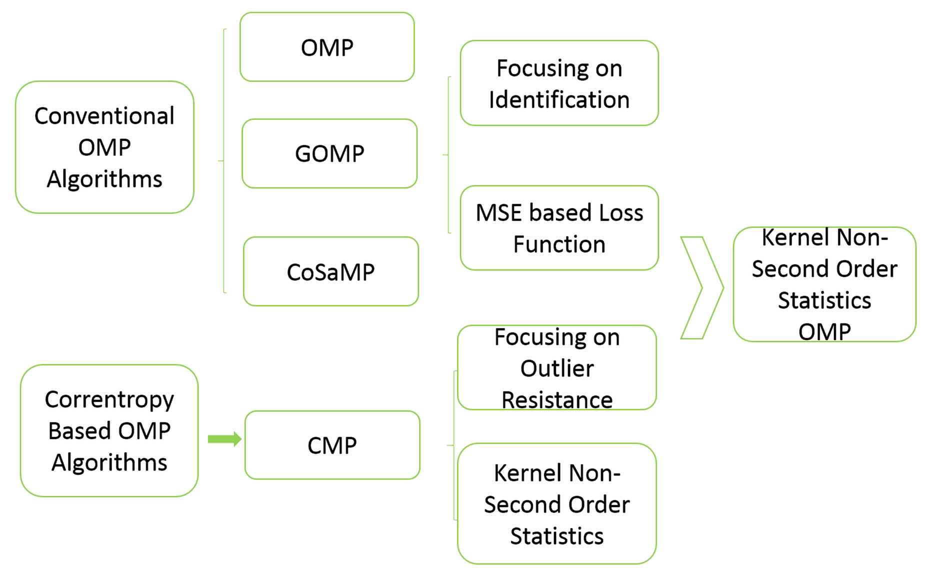
Abstract

Most matching pursuit algorithms are based on the mean square error(MSE) to minimize the recovery error, which is suboptimal when there are outliers. We present a new robust OMP algorithm based on kernel non-second order statistics (KNS-OMP), which not only takes advantages of the outlier resistance ability of correntropy but also further extends the second order statistics based correntropy to a non-second order similarity measurement to improve its robustness. The resulted framework is more accurate than the second order ones in reducing the effect of outliers.

Motivation



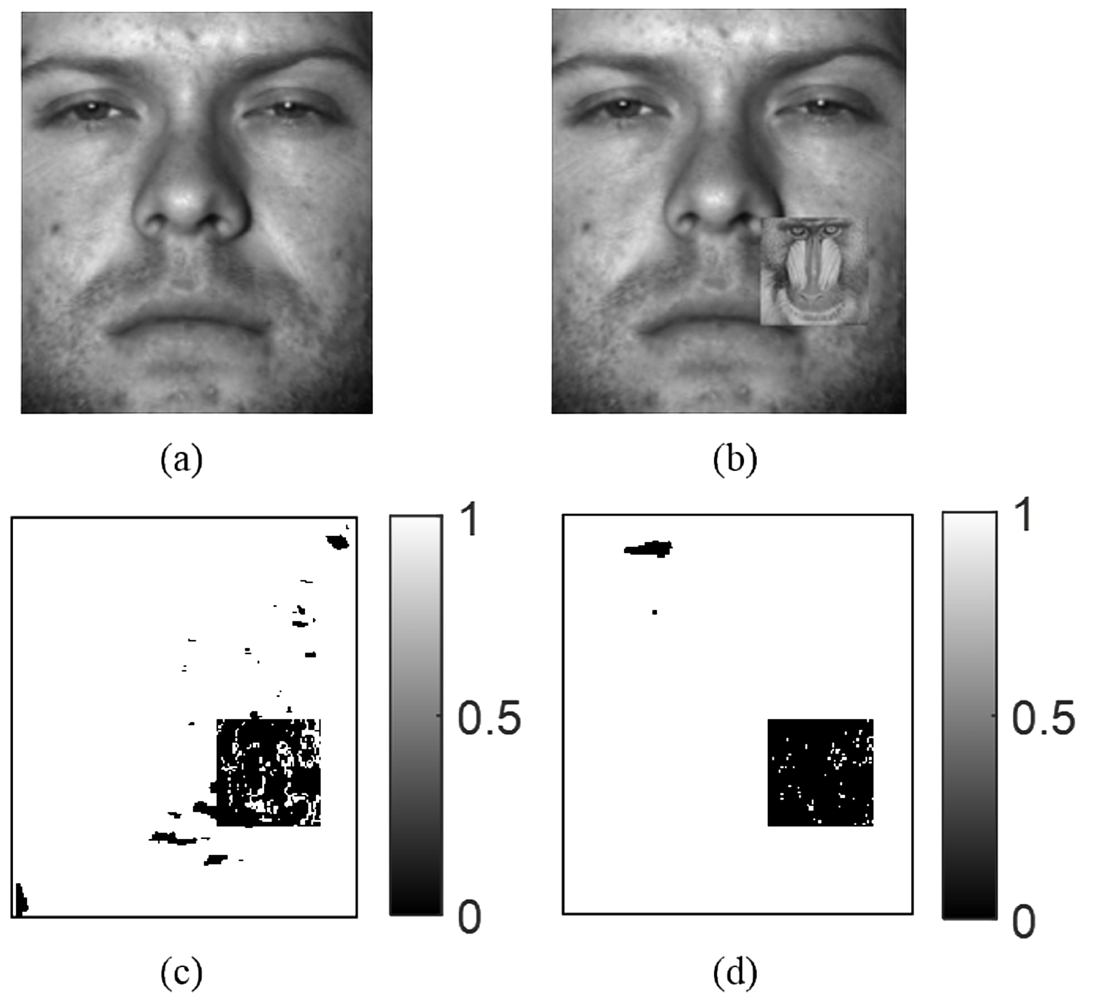


Figure: Weight images from CMP and the proposed algorithm. (a) Original image, (b) occluded image, (c) Weight images of CMP, (d) weight images of the proposed image.

Proposed Framework

The kernel non-second order loss function (KNS-loss) is defined as

$$J_{\text{KNS-loss}}(A,B) = 2^{-p/2}E[\| \varphi(A)-\varphi(B) \|^p_{\mathcal{H}}] = E[(1-g_\sigma(A-B))^{p/2}], \tag{1}$$

where p (>0) can be flexibly adjusted to improve the performance of sparse recovery. Then we obtain a new approximation $\mathbf{x}_k$ by solving the kernel non-second order loss function

$$J_{\text{KNS-loss}}(\mathbf{x}) = \frac{1}{m}\sum_{j=1}^{m}(1-g_\sigma(\mathbf{y}_i - \sum_{i=1}^{n}\mathbf{a}_{ij}\mathbf{x}_i))^{\frac{p}{2}} = \frac{1}{m}\sum_{i=1}^{m}\rho(||\mathbf{e}_i||_2), \tag{2}$$

According to the M-estimator and the definition of reweighted least squares, the objective is given by

$$\min \sum_{i=1}^{N}\gamma_i \mathbf{e}_i^2, \tag{3}$$

where $\mathbf{e}_i = \mathbf{y}_i - \sum_{i=1}^{n}\mathbf{a}_{ij}\mathbf{x}_i$ and function $\gamma(\mathbf{e}_i)$ is defined by $\gamma_i = \rho'(\mathbf{e}_i)/\mathbf{e}_i$. Then we have

$$\gamma_i^{k+1} = \frac{p}{2\sigma^2}\left[1-\exp\left(-\frac{\| \mathbf{e}_i \|_2^2}{2\sigma^2}\right)\right]^{\frac{p}{2}-1}\exp\left(-\frac{\| \mathbf{e}_i \|_2^2}{2\sigma^2}\right). \tag{4}$$

After obtaining the weights from (4), the sparse vector can be updated by

$$\mathbf{x}^{k+1} = \underset{\mathbf{x}\in R^n, \text{supp}(\mathbf{x})\subset S_k}{\text{argmin}} \left\| \sqrt{\text{diag}(\gamma^{k+1})}(\mathbf{y}-\mathbf{A}\mathbf{x})\right\|_2^2 \tag{5}$$

After obtaining the sparse vector, we update the residual by

$$\mathbf{r}^{k+1} = \sqrt{\text{diag}(\gamma^{k+1})}(\mathbf{y}-\mathbf{A}\mathbf{x}^{k+1}). \tag{6}$$

The kernel size σ can be updated by

$$\sigma^{k+1} = \sqrt{\frac{1}{2m}\|\mathbf{y}-\mathbf{A}\mathbf{x}^{k+1}\|_2^2}, \tag{7}$$

Experimental Results

Table: Average recovery error of different MP algorithms with various type of noise.Best results are marked in bold.

	$\chi^2(1)$	Exp.	Stud.	Miss.	Gau.	White
OMP	11.91	12.60	14.80	0.45	5.04	4.51
GOMP	12.55	11.90	14.68	0.58	5.09	4.38
CoSaMP	15.88	12.78	17.56	0.61	4.92	6.23
CMP	6.12	9.00	11.25	0.0133	5.22	6.21
Proposed	**5.89**	**7.97**	**10.85**	**0.0015**	**4.73**	**3.96**

Table: Average reconstruction error of different MP algorithms. RE1: random noise occlusion, RE2: real noise occlusion.

Methods	RE1	RE2
OMP	3.0023	3.3787
GOMP	3.3389	3.3872
CoSaMP	10.7602	20.7146
CMP	2.2025	2.5580
KNS-OMP ($p=1.5$)	**1.2078e-05**	0.0227
KNS-OMP ($p=1.7$)	0.0043	**0.0158**

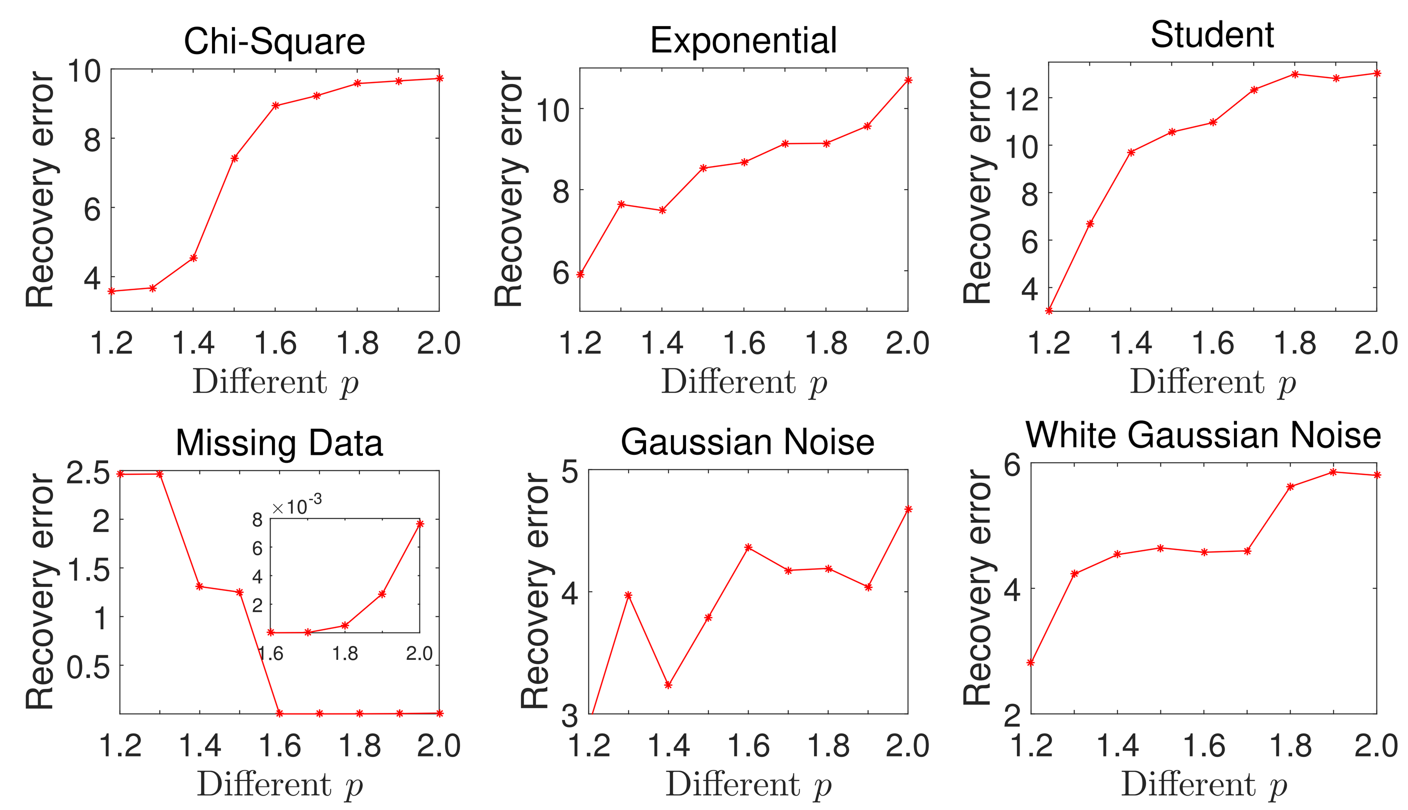


Figure: Average recovery error of the KNS-OMP with different p values with different types of noise.

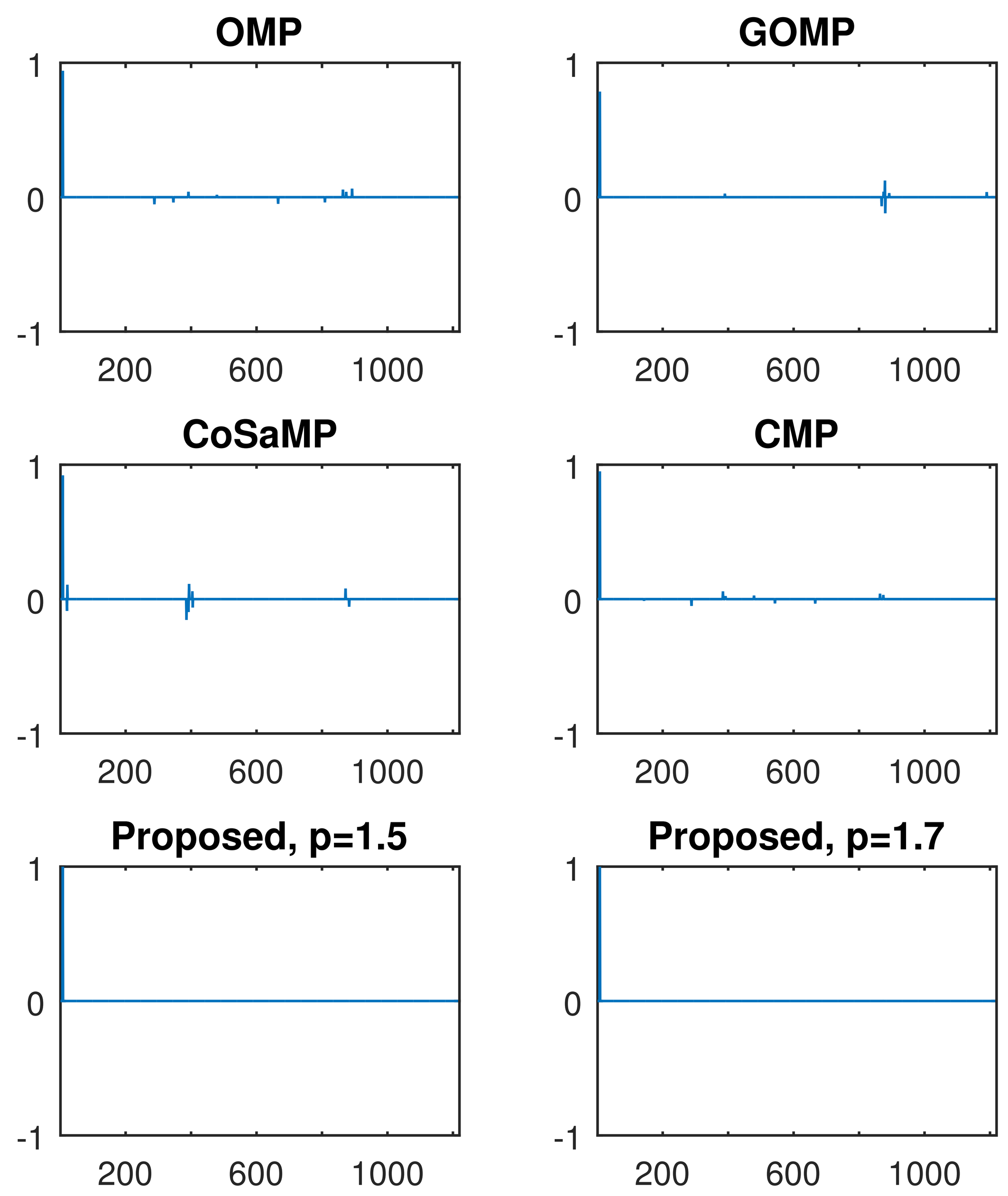


Figure: Sparse vector of different MP algorithms.

Conclusion

- Develop a non-second order statistics-based loss function to enhance the robustness of performance of the orthogonal pursuit algorithm.
- Result in greater noise resistance and occlusion detection ability and improved performances in applications of signal recovery and image reconstruction.

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