Parameter Identifiability of SS-Based Bistatic MIMO Radar (4650)

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Contribution: Establish more accurate conditions

Diversity smoothing has been widely developed for angle estimation with bistatic multiple input multiple output (MIMO) radar in the presence of coherent targets, the parameter identifiability of which is an important issue. In this paper, we are devoted to establishing more accurate conditions by studying the positive definiteness of smoothed target covariance matrix. The antenna numbers of transmit and receive arrays are derived as functions of the target number and target structure. We show that the new results improve upon previous ones and recover them in special cases. Simulation results are presented that corroborate our theoretical findings.

Abstract

Multiple input multiple output (MIMO) radar has been widely used for angle estimation due to the increased degrees of freedom (DOFs) and enhanced spatial resolution compared with the phased-array radar.

Different techniques have been proposed for target localization in the colocated MIMO radar, using the subspace-based methods such as multiple signal classification (MUSIC) and estimation of signal parameters via rotational invariance technique (ESPRIT). In the presence of coherent or correlated targets, however, the target covariance matrix is singular and these methods fail to resolve these targets. As a standard pre-processing technique, spatial smoothing (SS) has been widely used to overcome the coherency in the target covariance matrix at a cost of a small effective array size. This technique has been applied for angle estimation with MIMO radar known as diversity smoothing (DS).

The parameter identifiability of DS-based MIMO radar have been derived for coherent targets, where the sufficient conditions for bistatic MIMO radar were presented. However, the target is single-structured and the forward backward case has not been addressed.

References

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Diversity Smoothing

The received signal at the l-th snapshot can be written as

$$\begin{aligned}\boldsymbol{r}(l) &= [\boldsymbol{a}_t(\phi_1)\otimes\boldsymbol{a}_r(\theta_1),\cdots,\boldsymbol{a}_t(\phi_K)\otimes\boldsymbol{a}_r(\theta_K)]\,\boldsymbol{C}\boldsymbol{s}(l)\\ &\quad+\boldsymbol{n}(l)\\ &=(\boldsymbol{A}_t\circ\boldsymbol{A}_r)\,\boldsymbol{C}\boldsymbol{s}(l)+\boldsymbol{n}(l),\end{aligned}\tag{2}$$

Thus, assuming the noise and targets uncorrelated with each other, we calculate the array covariance matrix as

$$\begin{aligned}\boldsymbol{R} &= \mathrm{E}\left[\boldsymbol{r}(l)\boldsymbol{r}^H(l)\right]\\ &=(\boldsymbol{A}_t\circ\boldsymbol{A}_r)\boldsymbol{C}\boldsymbol{R}_s\boldsymbol{C}^H(\boldsymbol{A}_t\circ\boldsymbol{A}_r)^H+\sigma_n^2\boldsymbol{I}_{MN}\\ &=\boldsymbol{A}\boldsymbol{R}_{ss}\boldsymbol{A}^H+\sigma_n^2\boldsymbol{I}_{MN}.\end{aligned}\tag{3}$$

Diversity smoothing has been an effective method to decorrelate the coherent signals. Then, the (m_2,n_2)-th sliding window (covariance submatrix) is given by

$$\begin{aligned}\boldsymbol{R}_{m_2n_2} =& \boldsymbol{A}_1\boldsymbol{\Phi}_t^{m_2-1}\boldsymbol{\Phi}_r^{n_2-1}\boldsymbol{R}_s\boldsymbol{\Phi}_t^{1-m_2}\boldsymbol{\Phi}_r^{1-n_2}\boldsymbol{A}_1^H\\ &+\sigma_n^2\boldsymbol{I}_{M_1N_1},\end{aligned}\tag{4}$$

Taking the average of $\boldsymbol{R}_{m_2n_2}$ over all (m_2,n_2), the forward only DS (FOSS) matrix is expressed by

$$\begin{aligned}\boldsymbol{R}_f &= \frac{1}{M_2N_2}\sum_{m_2=1}^{M_2}\sum_{n_2=1}^{N_2}\boldsymbol{R}_{m_2n_2}\\ &=\frac{1}{M_2N_2}\boldsymbol{A}_1\boldsymbol{\Sigma}_f\boldsymbol{A}_1^H+\sigma_n^2\boldsymbol{I}_{M_1N_1},\end{aligned}\tag{5}$$

Likewise, the forward backward DS (FBDS) matrix is given by

$$\begin{aligned}\boldsymbol{R}_{fb} &= \frac{1}{2}\left(\boldsymbol{R}_f+\boldsymbol{J}_{M_1N_1}\boldsymbol{R}_f^*\boldsymbol{J}_{M_1N_1}\right)\\ &=\frac{1}{2}\boldsymbol{A}_1\boldsymbol{\Sigma}_{fb}\boldsymbol{A}_1^H+\sigma_n^2\boldsymbol{I}_{M_1N_1},\end{aligned}\tag{6}$$

Positive Definiteness and Parameter Identifiability

Lemma 1: The smoothed array covariance matrices $\boldsymbol{\Sigma}_f$ and $\boldsymbol{\Sigma}_{fb}$ are derived as in (4) and (5). $\mathrm{rank}(\boldsymbol{A})$ denotes the rank of $\boldsymbol{A}$. Then,

1. $\boldsymbol{\Sigma}_f$ is positive definite, if $\mathrm{rank}(\boldsymbol{A}_2^p)=\tilde{K}_p$;
2. $\boldsymbol{\Sigma}_{fb}$ is positive definite, if $\mathrm{rank}\left(\tilde{\boldsymbol{A}}_2^p\right)=\tilde{K}_p$, where $\tilde{\boldsymbol{A}}_2^p=\left[(\boldsymbol{A}_2^p)^T,(\boldsymbol{A}_2^p\boldsymbol{\Phi}_p)^T\right]^T$.

Remark 1: The study for the positive definiteness of the forward smoothed matrix $\boldsymbol{\Sigma}_f$ can be interpreted as a significant extension of the methods in [1-2] for the coherent and correlated targets. In particular, the proof of the forward backward smoothed covariance matrix $\boldsymbol{\Sigma}_{fb}$ using the linear independent concept and, more importantly, it is different from the quadratic product method. Besides, compared with the discussions in [2] which just consider the case of $K_1=K$, the proposed results are generalized and easily analyzed.

FOSS: Since $\mathrm{D}(\boldsymbol{A}_2^p)=\mathrm{D}(\boldsymbol{A}_2)$, we have $\mathrm{D}(\boldsymbol{A}_1)\geq K+1$, $\mathrm{D}(\boldsymbol{A}_2)\geq \max_p\tilde{K}_p$.
Assuming $M_1\geq N_1$ for brevity, the conditions can be revised as $M_1(N_1-1)\geq K, M_2N_2\geq\max_p\tilde{K}_p$, which satisfy

$$\begin{aligned}f_b &\triangleq M_1N_1+M_2N_2-M_1\\ &=MN-(M_1-1)N_2-(M_2-1)(N_1-1)\\ &\geq K+\max_p\tilde{K}_p.\end{aligned}\tag{1}$$

FBSS: Similarly, from *Lemma 1(b)*, the conditions are rewritten as $\mathrm{D}(\boldsymbol{A}_1)\geq K+1$, $\mathrm{D}\left(\tilde{\boldsymbol{A}}_2\right)\geq\max_p\tilde{K}_p$. We assume $M_1>M_2$ or $N_1>N_2$ as *a prior*, and we have $\mathrm{D}\left(\tilde{\boldsymbol{A}}_2\right)=2\mathrm{D}(\boldsymbol{A}_2)=2M_2N_2$. We get $M_1(N_1-1)\geq K$, $2M_2N_2\geq\max_p\tilde{K}_p$, which yield $f_b\geq K+\frac{\max_p\tilde{K}_p}{2}$.

Simulation Results

Fig.8

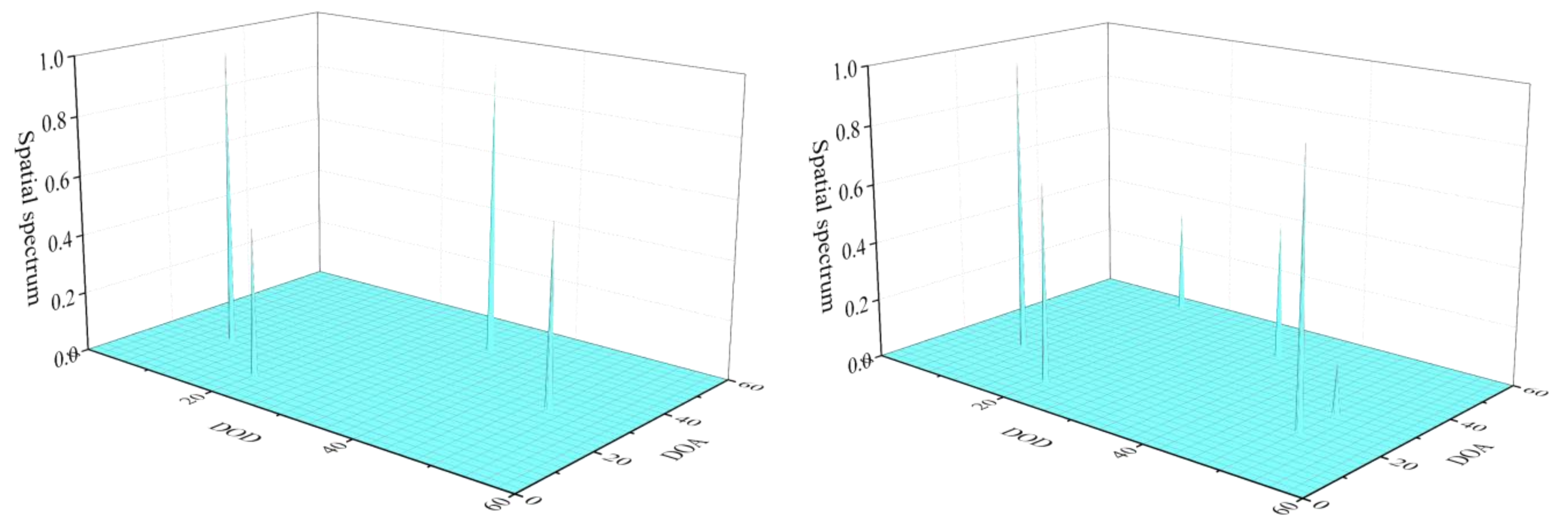

Fig.8 plots the maximum number of detectable targets for FODS and FBDS, where SNR is 60dB and the number of snapshots is set as 500. As shown, FODS identifies at most $f_b-r=4$ targets with $M_1=3, N_1=2$, and FBDS resolves up to $f_b-r/2=6$ targets with $M_1=N_1=3$. As a result, FBDS detects more targets than FODS for the bistatic MIMO radar.
Moreover, according to the previous result, FODS can identify 3 targets with $M_1=N_1=2$, while Fig.8 has shown that 4 targets can be detected. It is obviously seen that the proposed result is more accurate than that of [2].