SYMMETRY-BASED GRAPH FOURIER TRANSFORMS FOR IMAGE REPRESENTATION

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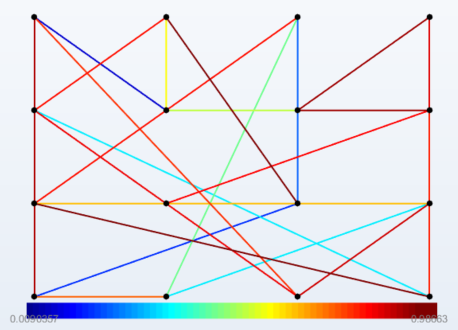
Overview

- In block-wise transform coding schemes
  - DCT: asymptotically equivalent to the KLT of a first order Markov process
- In this work: proposal for a set of Symmetry-Based Graph Fourier Transforms (SBGFTs)
  - Totally or partially symmetric grid
- Approximation ability analysis
  - Natural images
  - Residual signals (taken from intra-prediction in HEVC)
- Results: the set of SBGFTs markedly outperforms the DCT → notable statistical matching with data

Preliminaries

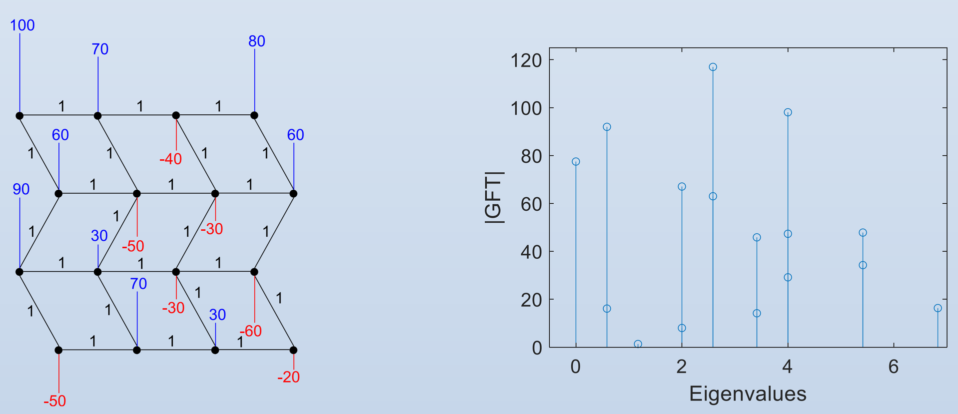
- In Graph Signal Processing, a signal f is defined on a graph $G = \{V, E, \boldsymbol{W}\}$
  - Set of vertices $V = \{1, 2, \dots, N\}$
  - Set of edges E
  - Weighted adjacency matrix

$$\boldsymbol{W} = \begin{cases} w_{ij} & if \quad \exists\, e(i,j) \\ 0 & if \quad \nexists\, e(i,j) \end{cases}$$



Example of graph.

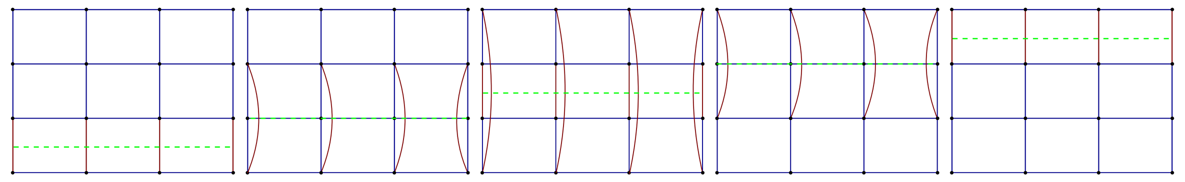
- Naming $\boldsymbol{D}$ the diagonal degree matrix of G
  - *Graph Laplacian matrix* $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{W}$
- Eigen-decomposition of $\boldsymbol{L}$ such that $\boldsymbol{L} = \boldsymbol{T}\boldsymbol{\Lambda}\boldsymbol{T}^{-1}$
- Graph Fourier Transform $F = \boldsymbol{T}f \leftrightarrow f = \boldsymbol{T}^{-1}F$



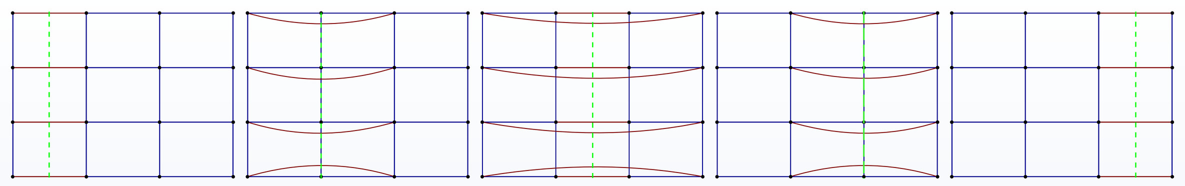
Signal on a graph (left) and the corresponding GFT in the graph spectral domain (right)

Symmetry-based GFT

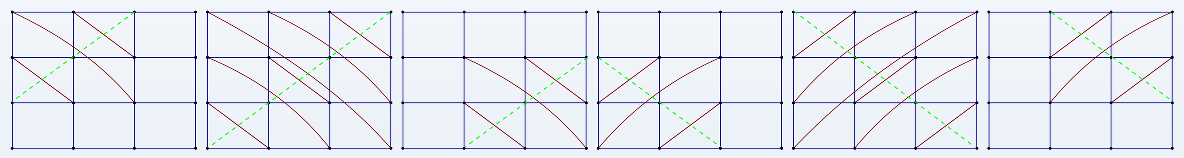
- Idea: set of graphs whose symmetric grids may well represent symmetries of a 2-D signal
- Which symmetries?
  - Symmetry axes
    - Slopes: 0°, 45°, 90°, 135°
    - Different offsets
  - Centro-symmetries
- Details
  - Preliminary analysis: $4x4$ grids
  - Starting configuration: 2-D grid with *low* weights
  - Edges are added so that nodes get connected to reflect the considered symmetry (*large* weights)



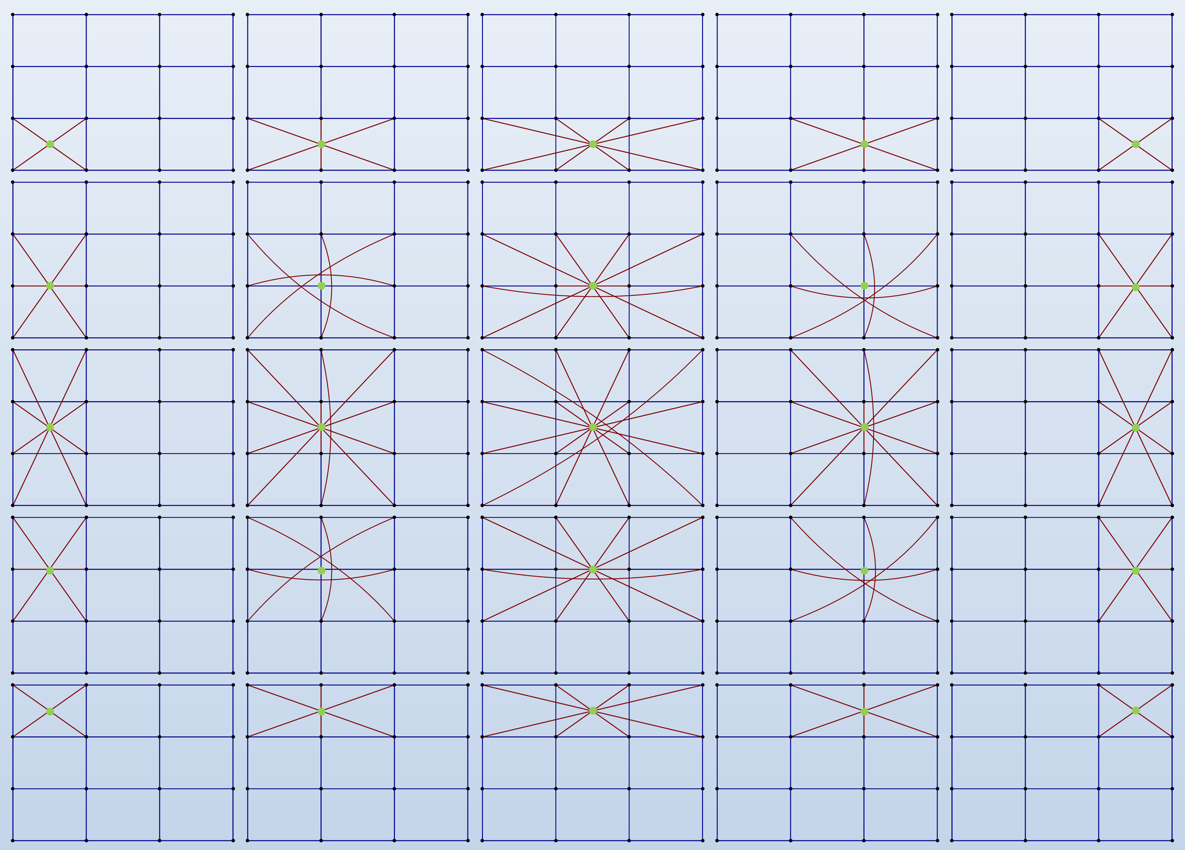
Five up-down (UD) symmetries



Five left-right (LR) symmetries



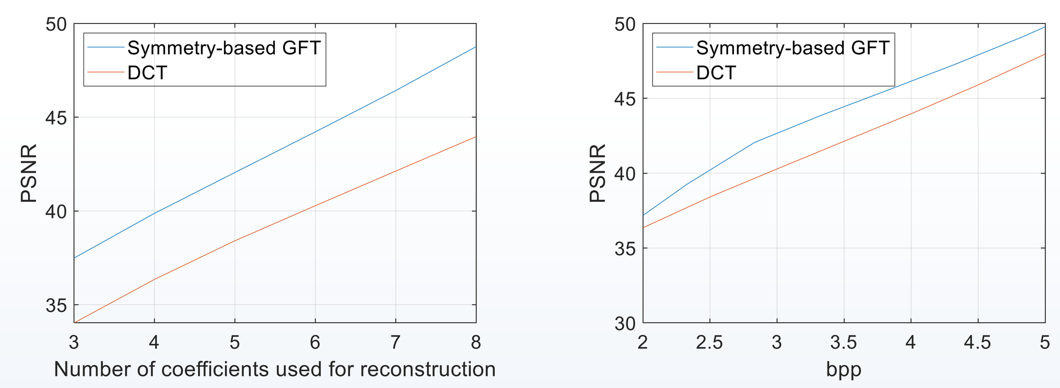
Three diagonal and three anti-diagonal symmetries



Twenty-five centro-symmetries

Representativeness of SBGFTs

- Evaluation of the energy compaction ability of the set of SBGFTs
- Experiments on USC-SIPI Image Database
- Brute-force strategy
  - For each $4x4$ image block all the 41 SBGFTs are tested
  - Non-linear approximation (keeping the K largest coefficients in absolute value)
  - A winner is set according to the SBGFT leading the smallest MSE
- Comparison with the DCT
  - PSNR-K curve (no signaling overhead)
  - PSNR-bpp curve
    - 8 bits per saved coefficient
    - $\lceil \log_2 41 \rceil = 6$ overhead bits (only SBGFT)



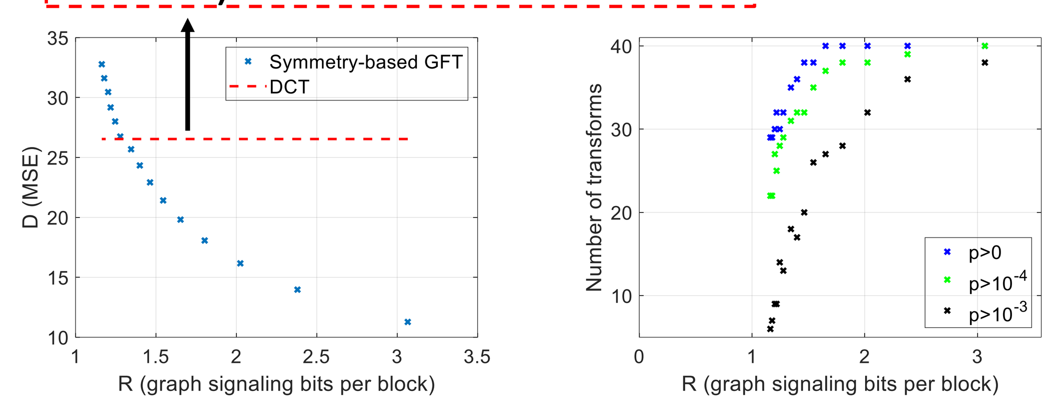
Performance SBGFT vs DCT

Experiments on residuals

- Dataset: $4x4$ residual blocks obtained during the intra-prediction in HEVC (four video sequences)
- Connection between prediction mode and optimal SBGFT?
- Block database divided in 35 classes
- Procedure similar to the previous one
  - For each $4x4$ residual block all the 41 SBGFTs are tested
  - Non-linear approximation (the $K = 4$ largest coefficients in modulus are kept)
  - The optimal SBGFT is chosen such that $min\{J\} \to J = D + \lambda R$

Graph index entropy

The MSE for DCT (independent from R) is shown for reference



Distortion (left) and number of transforms (right) *vs* graph index entropy (horizontal prediction)

- Performance consistent for each mode
- The entropic rates (related to the signaling overhead) are considerably lower than the previously considered 6 bits per block
- Cardinality of the set can be reduced removing irrelevant transforms (based on the examined prediction mode)

Future (current) work

- Extension to $8x8$ symmetric grids
  - Constraint: choose the associated SBGFTs so that fast implementations exist
- Simulation of a complete image/video coder
  - Uniform quantization with dead-zone instead of non-linear approximation
  - $min\{J\} \to J = D + \lambda(R_G + R_c)$

Graph index entropy

Non-zero coefficient (location and level) entropy

  - R_c ad hoc for each graph
- Comparison with *JPEG, JPEG2000, intra-prediction followed by DCT (HEVC), …*
  - Promising performance
- Next:
  - Methods to speed up the graph choice
  - Graph learning with respect to the weights (fixed topology)
  - Better understanding of the relation between data (before and after quantization) and graphs