A Data-Selective LS Solution to TDOA-based Source Localization

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Objectives
- Review the classical LS solution of the TDOA-based source localization problem
- Employ data-selection to this solution
- Show the performance of the new approach
- Point out scenarios for this new solution

Introduction
- The source localization problem has applications in many fields
  - Usual strategies: received signal strength (RSS), direction of arrival (DOA), time of arrival (TOA), and time difference of arrival (TDOA)
- The TDOA strategy measures the difference of transmission time of a single signal between the receiver nodes
  - It has a closed form, is more robust to reflections and easier to implement (lower computational burden and requiring no sensor arrays)
- The signal from the sensors must be synchronized

The Classical Approach
- In a 2D scenario, \( M \) sensors with known positions given by \( \mathbf{p}_m, 1 \leq m \leq M \Rightarrow N = \frac{M(M-1)}{2} \) TDOAs
- We define the range-difference from the unknown source and sensors \( i \) and \( j \):
  \[ \Delta d_{ij} = d_{ij} - d_{ji} \quad i > j \]
- The TDOA \( \tau_{ij} \) (in number of samples) is obtained from the peak of the cross-correlation of the signals acquired by the sensors
- For the \( m \)-th sensor, \( d_{im} \) is the distance from the source (unknown position \( \mathbf{p} \)) to the \( m \)-th sensor
  - Therefore, we can write \( \| \mathbf{p} - \mathbf{p}_m \| = d_{im} \) and \( \| \mathbf{p} - \mathbf{p}_m \|^2 = (d_{im} + \Delta d_{im})^2 \cdot \cdot \cdot \)
  - \( \cdot \cdot \cdot \) which leads to \( (\mathbf{p} - \mathbf{p}_m) \mathbf{T} \mathbf{p} + \Delta d_{im} d_{im} = b_{im} \), where \( b_{im} = \frac{\| \mathbf{p} - \mathbf{p}_m \|^2 - \Delta d_{im}^2}{2}, \cdot \cdot \cdot \leq m \leq M \)

Using more TDOAs
- The previous solution uses only \( M-1 \) from a total of \( N = \frac{M(M-1)}{2} \) TDOA measurements
- Assuming similar errors, we expect that using more measurements leads to more accurate results
- Using \( \| \mathbf{p} - \mathbf{p}_m \|^2 = (d_{im} + \Delta d_{im})^2 \) instead of \( d_{im} \):
  \[ \xi_2 = \sum_{m=1}^{M}(\| \mathbf{p} - \mathbf{p}_m \|^2 - \Delta d_{im} d_{im} \cdot \cdot \cdot \leq m \leq M \)
- \( \Delta \mathbf{d}_i \) and the \( (M-2) \times 1 \) error vector \( \mathbf{e}_2 \) is given as:
  \[ \mathbf{e}_2 = \begin{bmatrix} (\mathbf{p}_1 - \mathbf{p}_2) \mathbf{T} \Delta d_{12} & \vdots & (\mathbf{p}_M - \mathbf{p}_2) \mathbf{T} \Delta d_{M2} \\ (\mathbf{p}_1 - \mathbf{p}_2) \mathbf{T} \Delta d_{12} & \vdots & (\mathbf{p}_M - \mathbf{p}_M) \mathbf{T} \Delta d_{MM} \end{bmatrix} \begin{bmatrix} d_{12} \\ \vdots \\ d_{MM} \end{bmatrix} \]
- Similarly, we do the same for \( d_{M-1} \) defining matrices \( \mathbf{A}_1 \) to \( \mathbf{A}_{M-1} \) and vectors \( \mathbf{b}_1 \) to \( \mathbf{b}_{M-1} \)

The Extended Solution
- The extended cost function is formed using all \( N \) TDOA measurements:
  \[ \xi = \sum_{m=1}^{M}(\mathbf{e}_m)^T \mathbf{e}_m \]
- From the definitions of \( \xi_m \), \( \mathbf{e}_m \) and \( \mathbf{A}_m \), the extended cost function can be expressed as: \( \xi = \mathbf{e}^T \mathbf{e}, \mathbf{e} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_M \end{bmatrix} \mathbf{e}^T = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_M \end{bmatrix} \begin{bmatrix} \mathbf{e}_1^T \\ \mathbf{e}_2^T \\ \vdots \\ \mathbf{e}_M^T \end{bmatrix} \]
- Such that the extended LS solution is given by:
  \[ \hat{\mathbf{p}} = \frac{1}{M} \mathbf{A}_1 \mathbf{A}_1^T \mathbf{A}_2 \mathbf{A}_2^T \cdots \mathbf{A}_M \mathbf{A}_M^T \mathbf{e} \]

The Proposed Algorithm
- Let \( \mathbf{p}_m, 1 \leq m \leq M \) be the positions of \( M \) sensors
- Let \( \tau_{ij} \) be the \( N \) available TDOAs (in \( \Delta \) samples)
- Set \( \nu \) and \( \kappa \), and choose \( n \)
- \( \Delta d_{ij} \leftarrow \nu \Delta d_{ij} \) for all \( N \) TDOAs
- Form matrix \( \mathbf{A} \) and vector \( \mathbf{b} \)
  \[ \mathbf{p}_n \leftarrow \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{A}^{-1} \mathbf{A}^T \mathbf{b} \]
- \( \Delta d_{ij} \leftarrow \nu \Delta d_{ij} \) for all \( 1 \leq m \leq M \) sensors
- \( \Delta d_{ij} \leftarrow \nu d_{ij} \) for all \( N \) possible pairs \( (i, j) \)
- \( \xi_{min} \leftarrow \min \left\{ (\Delta d_{ij} - \Delta d_{ij})^2 \right\} \) for each subset of measurements \( \mathbf{S}_n \)
- \( \mathbf{p}_n \leftarrow \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{A}^{-1} \mathbf{A}^T \mathbf{b} \)
- \( \Delta d_{ij} \leftarrow \nu \Delta d_{ij} \) for all \( 1 \leq m \leq M \) sensors
- \( \Delta d_{ij} \leftarrow \nu d_{ij} \) for all \( N \) possible pairs \( (i, j) \)
- \( \xi_{min} \leftarrow \min \left\{ (\Delta d_{ij} - \Delta d_{ij})^2 \right\} \)
- \( \mathbf{p}_n \leftarrow \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{A}^{-1} \mathbf{A}^T \mathbf{b} \)

Key Concept
- We next extend the number of pairs from \( (M-1) \) to \( M(M-1)/2 \) (as in [Khalaf-Allah, 2014]) and, assuming a number of incorrect TDOAs estimates, propose a criterion to select those leading to better results!

Experimental Results

<table>
<thead>
<tr>
<th>T/R</th>
<th># outliers</th>
<th>Geometry</th>
<th>Conv. LS</th>
<th>Est. LS</th>
<th>18S-LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>10dB/3</td>
<td>1</td>
<td>4.1485</td>
<td>4.0010</td>
<td>1.3808</td>
<td></td>
</tr>
<tr>
<td>20dB/2</td>
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<td>2.2890</td>
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<td>0.8884</td>
<td>0.9319</td>
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<tr>
<td>40dB/0</td>
<td>2</td>
<td>0.0140</td>
<td>0.0098</td>
<td>0.0010</td>
<td></td>
</tr>
</tbody>
</table>

Conclusions
- DS is effective in TDOA-based localization
- The algorithm is more successful for noisy and reverberant scenarios