A Data-Selective LS Solution to TDOA-based Source Localization

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Objectives

- Review the classical LS solution of the TDOA-based source localization problem
- Employ data-selection to this solution
- Show the performance of the new approach
- Point out scenarios for this new solution

Introduction

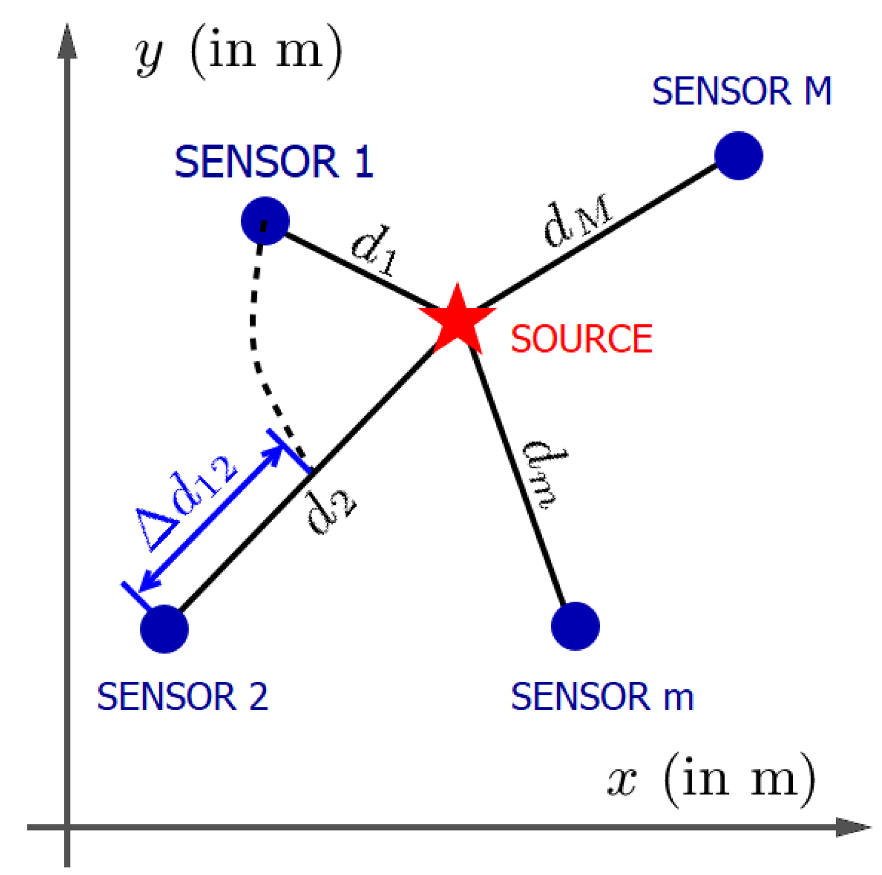
- The source localization problem has applications in many fields
- Usual strategies: received signal strength (RSS), direction of arrival (DOA), time of arrival (TOA), and time difference of arrival (TDOA)
- The TDOA strategy measures the difference of transmission time of a single signal between the receiver nodes
- It has a closed form, is more robust to reflections and easier to implement (lower computational burden and requiring no sensor arrays)
- The signal from the sensors must be synchronized

The Classical Approach

- In a 2D scenario, M sensors with known positions given by $\mathbf{p}_m$, $1 \leq m \leq M \Rightarrow N = \frac{M(M-1)}{2}$ TDOAs
- We define the range-difference from the unknown source and sensors i and j: $\Delta d_{ij} = d_i - d_j = \frac{v\tau_{ij}}{f_s}$, $i > j$
- The TDOA τ_{ij} (in number of samples) is obtained from the peak of the cross-correlation of the signals acquired by the sensors
- For the m-th sensor, d_m is the distance from the source (unknown position $\mathbf{p}$) to the m-th sensor
- Therefore, we can write $\|\mathbf{p}-\mathbf{p}_1\|^2 = d_1^2$ and $\|\mathbf{p}-\mathbf{p}_m\|^2 = (d_1 + \Delta d_{m1})^2 \cdots$
- $\cdots$ which leads to $(\mathbf{p}_m - \mathbf{p}_1)^{\mathrm{T}}\mathbf{p} + \Delta d_{m1} d_1 = b_{1m}$, where $b_{1m} = \frac{\|\mathbf{p}_m\|^2 - \|\mathbf{p}_1\|^2 - \Delta d_{m1}^2}{2}$, $2 \leq m \leq M$

The LS Cost Function

$$\xi_1 = \sum_{m=2}^{M}\left[\left((\mathbf{p}_m - \mathbf{p}_1)^{\mathrm{T}}\mathbf{p} + \Delta d_{m1} d_1 - b_{1m}\right]^2\right.$$



The closed-form solution

- ξ_1 can be expressed $\|\mathbf{e}_1\|^2$ where

$$\mathbf{e}_1 = \underbrace{\begin{bmatrix} (\mathbf{p}_2-\mathbf{p}_1)^{\mathrm{T}} & \Delta d_{21} \\ (\mathbf{p}_3-\mathbf{p}_1)^{\mathrm{T}} & \Delta d_{31} \\ \vdots & \vdots \\ (\mathbf{p}_M-\mathbf{p}_1)^{\mathrm{T}} & \Delta d_{M1} \end{bmatrix}}_{\mathbf{A}_1} \underbrace{\begin{bmatrix} \mathbf{p} \\ d_1 \end{bmatrix}}_{\mathbf{x}_1} - \underbrace{\begin{bmatrix} b_{12} \\ b_{13} \\ \vdots \\ b_{1M} \end{bmatrix}}_{\mathbf{b}_1}$$

- Note (in a 2D scenario) that $\mathbf{A}_1$ is an $(M-1) \times 3$ matrix $\rightsquigarrow$ if we have at least four sensors, an unconstrained solution is obtained after $\nabla_{\mathbf{x}_1}\xi_1 = \mathbf{0}$
- The estimated position of the source is given by

$$\hat{\mathbf{p}} = [\mathbf{I}\ \ \mathbf{0}]\,\hat{\mathbf{x}}_1 = [\mathbf{I}\ \ \mathbf{0}]\left(\mathbf{A}_1^{\mathrm{T}}\mathbf{A}_1\right)^{-1}\mathbf{A}_1^{\mathrm{T}}\mathbf{b}_1$$

Key Concept

We next extend the number of pairs from $(M-1)$ to $\frac{M(M-1)}{2}$ (as in [Khalaf-Allah, 2014]) and, assuming a number of incorrect TDOAs estimates, propose a criterion to select those leading to better results!

Using more TDOAs

- The previous solution uses only $M-1$ from a total of $N = \frac{M(M-1)}{2}$ TDOA measurements
- Assuming similar errors, we expect that using more measurements leads to more accurate results
- Using $\|\mathbf{p}-\mathbf{p}_m\|^2 = (d_2 + \Delta d_{m2})^2$ instead of d_1:

$$\xi_2 = \sum_{m=3}^{M}\left((\mathbf{p}_m - \mathbf{p}_2)^{\mathrm{T}}\mathbf{p} + \Delta d_{m2} d_2 - b_{2m}\right)^2 = \|\mathbf{e}_2\|^2$$

where $b_{2m} = \frac{\|\mathbf{p}_m\|^2 - \|\mathbf{p}_2\|^2 - \Delta d_{m2}^2}{2}$, $3 \leq m \leq M$

- $\cdots$ and the $(M-2) \times 1$ error vector $\mathbf{e}_2$ is given as

$$\mathbf{e}_2 = \underbrace{\begin{bmatrix} (\mathbf{p}_3-\mathbf{p}_2)^{\mathrm{T}} & \Delta d_{32} \\ \vdots & \vdots \\ (\mathbf{p}_M-\mathbf{p}_2)^{\mathrm{T}} & \Delta d_{M2} \end{bmatrix}}_{\mathbf{A}_2} \underbrace{\begin{bmatrix} \mathbf{p} \\ d_2 \end{bmatrix}}_{\mathbf{x}_2} - \underbrace{\begin{bmatrix} b_{23} \\ \vdots \\ b_{2M} \end{bmatrix}}_{\mathbf{b}_2}$$

- Similarly, we do the same for d_3 to d_{M-1} defining matrices $\mathbf{A}_3$ to $\mathbf{A}_{M-1}$ and vectors $\mathbf{b}_3$ to $\mathbf{b}_{M-1}$

The Extended Solution

- The extended cost function is formed using all N TDOAs measurements:

$$\xi = \sum_{m=1}^{M}\xi_m,$$

where $\xi_m = \mathbf{e}_m^{\mathrm{T}}\mathbf{e}_m$, and $\mathbf{e}_m = \mathbf{A}_m[\mathbf{p}^{\mathrm{T}}\ d_m]^{\mathrm{T}} - \mathbf{b}_m$

- From the definitions of ξ_m, $\mathbf{e}_m$ and $\mathbf{A}_m$, the extended cost function can be expressed as $\xi = \mathbf{e}^{\mathrm{T}}\mathbf{e}$, where

$$\underbrace{\begin{bmatrix} (\mathbf{p}_2-\mathbf{p}_1)^{\mathrm{T}} & \Delta d_{21} & 0 & 0\cdots 0 \\ \vdots & \vdots & \vdots & \vdots\ \vdots \\ (\mathbf{p}_M-\mathbf{p}_1)^{\mathrm{T}} & \Delta d_{M1} & 0 & 0\cdots 0 \\ (\mathbf{p}_3-\mathbf{p}_2)^{\mathrm{T}} & 0 & \Delta d_{32} & 0\cdots 0 \\ \vdots & \vdots & \vdots & \vdots\ \vdots \\ (\mathbf{p}_M-\mathbf{p}_2)^{\mathrm{T}} & 0 & \Delta d_{M2} & 0\cdots 0 \\ & \vdots & & \vdots \\ (\mathbf{p}_M-\mathbf{p}_{M-1})^{\mathrm{T}} & 0 & 0 & \cdots 0\Delta d_{M(M-1)} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{p} \\ d_1 \\ d_2 \\ \vdots \\ d_{M-1} \end{bmatrix}}_{\mathbf{x}} - \underbrace{\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_{M-1} \end{bmatrix}}_{\mathbf{b}}$$

- Such that the extended LS solution is given by

$$[\hat{\mathbf{p}}^{\mathrm{T}}\ \hat{d}_1\ \hat{d}_2\ \cdots\ \hat{d}_{M-1}]^{\mathrm{T}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

The Proposed Algorithm

Let $\mathbf{p}_m$, $1 \leq m \leq M$, be the positions of M sensors
$N \leftarrow \frac{M(M-1)}{2}$
Let τ_{ij} be the N available TDOAs (in # samples)
Set v and f_s, **and choose** n
$\Delta d_{ij} \leftarrow \frac{v\tau_{ij}}{f_s}$ for all N TDOAs
$|\Delta d_{ij}|_{\max} \leftarrow \|\mathbf{p}_i - \mathbf{p}_j\|$ for all N TDOAs
Form matrix $\mathbf{A}$ and vector $\mathbf{b}$
$\hat{\mathbf{p}}_n \leftarrow [\mathbf{I}\ \ \mathbf{0}]\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$
$\hat{d}_m \leftarrow \|\hat{\mathbf{p}}_n - \mathbf{p}_m\|$ for all $1 \leq m \leq M$ sensors
$\Delta\hat{d}_{ij} \leftarrow \hat{d}_i - \hat{d}_j$ for all N possible pairs $\{i,j\}$
$\xi_{min} \leftarrow \frac{1}{N}\sum_{\{i,j\}}\left(\Delta\hat{d}_{ij} - \Delta d_{ij}\right)^2$
for each subset of measurements $\mathcal{S}_n$
do
- **if** all $\{|\Delta d_{ij}| \leq |\Delta d_{ij}|_{\max}, \{i,j\} \in \mathcal{S}_n\}$
- **then**
  - **Adjust** $\mathbf{A}_n$ and $\mathbf{b}_n$ according to $\mathcal{S}_n$
  - $\hat{\mathbf{p}}_n \leftarrow [\mathbf{I}\ \ \mathbf{0}]\left(\mathbf{A}_n^{\mathrm{T}}\mathbf{A}_n\right)^{-1}\mathbf{A}_n^{\mathrm{T}}\mathbf{b}_n$
  - $\hat{d}_m \leftarrow \|\hat{\mathbf{p}}_n - \mathbf{p}_m\|$ for $1 \leq m \leq M$
  - $\Delta\hat{d}_{ij} \leftarrow \hat{d}_i - \hat{d}_j$ for $\{i,j\} \in \mathcal{S}_n$
  - $\xi_n \leftarrow \frac{1}{n}\sum_{\{i,j\}\in\mathcal{S}_n}\left(\Delta\hat{d}_{ij} - \Delta d_{ij}\right)^2$
  - **if** $\xi_n < \xi_{min}$
  - **then** $\xi_{min} \leftarrow \xi_n$; $\mathbf{p}_o \leftarrow \hat{\mathbf{p}}_n$

return $(\mathbf{p}_o)$

Experimental Results

TNR / # outliers	Geometry	Conv. LS	Ext. LS	DS-LS
10dB / 3	1	4.1885	4.0103	**1.3808**
	2	5.4328	3.5410	**0.3731**
20dB / 2	1	2.1831	2.2836	**0.1312**
	2	2.2328	1.7517	**0.1073**
30dB / 1	1	0.8383	0.9316	**0.0355**
	2	0.5861	0.5022	**0.0337**
40dB / 0	1	0.0140	**0.0095**	0.0110
	2	0.0108	**0.0065**	0.0108

$$TNR = 10\log\left(\frac{\frac{1}{N}\sum_{ij}\tau_{ij}^2}{\sigma_{TDOAerror}^2}\right)$$

Conclusions

- DS is effective in TDOA-based localization
- The algorithm is more successful for noisy and reverberant scenarios

ICASSP 2019
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