Introduction
Consider a sampling problem, where a spatiotemporal field governed by a linear constant coefficient partial differential equation (PDE) is sampled by a mobile sensor. Contrast the following:

Classical: uniform sampling with known timestamps
Our work: both the location and timestamps of mobile sensor are unknown
The sampling process can be depicted via the following plots:

Analytical setup
Sampling location and sampling time model
Sampling points form two independent unknown renewal processes in space and time
The mean of \( M \) is the average sampling density, i.e., \( n \)

In details:
- Sensing starts at \( x = 0, t = 0 \) ends before \( x = 1, t = T_f \)
- The intersample distances are a realization of an unknown renewal process
- The intersample times are a realization of another independent unknown renewal process

Distortion criterion
For any estimate of the field \( \hat{G}(x, t) \)

\[
\mathbb{E} \left[ \left| \hat{G}(x, t) - g(x, t) \right|^2 \right] \leq \mathbb{E} \left[ \sum_{k=-b}^{b} \left| \hat{A}_k - a_k(0) \right|^2 \right]
\]

Main Result
Theorem: Let \( \hat{A}_k(0), - b \leq k \leq b \) be the output of our interference algorithm. Then, the mean-squared error and therefore the distortion \( D \) is bounded as

\[
\mathbb{E} \left[ \sum_{k=-b}^{b} \left| \hat{A}_k(0) - a_k(0) \right|^2 \right] \leq O(1/n)
\]

The key result of our paper as well as its challenge is an analytical proof of the above

Theorem’s illustration:
The field’s bandwidth is \( b = 3 \), and its coefficients at \( t = 0 \) are generated from uniformly distributed random variables. The diffusion equation was used as PDE:

\[
\frac{\partial}{\partial t} g(x, t) = 0.01 \frac{\partial^2}{\partial x^2} g(x, t)
\]

Related works
Mobile sensing:
- Unnikrishnan and Vetterli’2013 (the idea of using a mobile sensor and associated aliasing/path density tradeoffs)
- Kumar’2017 (location-unaware mobile sensing using temporally fixed fields)

Ongoing developments in location-unaware sensing:
- Kumar’2015 (scattered location-unaware sensors and associated results)
- Pacholska, Haro, Schoelefetter, Vetterli’2017 (uniqueness constraints for ensuring field reconstruction)
- SLAM algorithm and its variants

Conclusions and future work
- Spatiotemporal and initially bandlimited fields evolving by a linear PDE can be reconstructed from location-unaware samples taken on an unknown renewal process
- The mean-squared error scales as \( O(1/n) \), where \( n \) the average number of samples
- The regression framework is universal, since neither it requires the renewal process distributions nor the noise distribution
- Exploration of two-dimensional spatial fields is of interest
- Exploration of approximately bandlimited field’s sampling and reconstruction is of interest

Motivation
Economic
A location-unaware and time-unaware mobile sensor will avoid the costs of GPS, other accurate localization mechanisms, and a precise clock

Social
A location-unaware sensor will preserve the privacy of the mobile sensor (assuming it is with a social device)

Academic
What is the fundamental impact of not knowing the sample locations in spatial field reconstruction problems?

Noise model
It is assumed that each measured sample \( g(x, t) \) is affected by an independent and identically distributed, zero-mean, finite variance noise \( \mathcal{N}(x, t) \)

Our inference algorithm
- \( g(x, t) \) can be written as an inner product, of Fourier basis dependent vectors and location dependent coefficients
- Using linearity in the Fourier coefficients, the problem is cast as a linear regression to estimate the Fourier coefficients at \( t = 0 \)
- Where \( g \) is a vector formed by measurement-noise affected samples, \( Y \) is a matrix formed by the location dependent vectors and \( b \) is the vector of target Fourier coefficients
- For regression, the (location-time) pair of samples are approximated as \( T_i \approx iT/M \) and \( S_j \approx j/M \)

Simulation
Three different PDEs were examined:
- \( g(x, 0) \) was generated by uniformly distributed Fourier series coefficients. The final field was scaled to ensure \( |g(x, 0)| \leq 1 \)
- The initial field evolved with the following PDEs:
  - Eq. 1, \( p = (2, 3, 0), q = (0.000125, 0, 0.01, 0, 0) \)
  - Eq. 2, \( p = (1, 3, 0), q = (0.01, 0, 0) \)
  - Eq. 3, \( p = (1, 0), q = (0.01, 0, 0) \)
- A benchmark was also used where locations of samples were random but known

- The renewal processes were generated using uniformly distributed random variables

- The number of samples

\[
\text{Number of samples} = \frac{1}{M^2}
\]