1 Introduction

We investigate the impact of the network configuration on the level of favorable propagation for a cell-free (CF) Massive MIMO network. Leveraging users’ spatial diversity, we formulate a user grouping and scheduling optimization problem. The formulated optimization problem is NP-hard. We design an efficient randomization algorithm based on semidefinite relaxation method to efficiently find a sub-optimal solution.

2 System Model

- $M$ single antenna APs serve simultaneously $N$ single omni-directional antenna users ($N < M$).
- The $m$-th AP performs minimum mean-square error (MMSE) channel estimation $\hat{h}_{mk} = \frac{1}{\sqrt{P_{\text{t},m}}} \sum_{k=1}^{K} \alpha_{mk} X_{mk} + n_{mk}$, $k = 1, \ldots, r$, where:
  - $\hat{h}_{mk}$: channel coefficient between the $k$-th user and the $m$-th AP.
  - $\alpha_{mk}$: training sequence of the $k$-th user.
  - $P_{\text{t},m}$: transmit power during training phase.
  - $n_{mk}$: AWGN vector at the $m$-th AP.
- $r$: uplink training duration with $r < T$, (coherence interval).

3 Which users can be active simultaneously?

Favorable propagation: mutual orthogonality between users vector wireless channel $\hat{h}_{mk} \sim \mathcal{C}\mathcal{N}(0, I_{N})$. Asymptotically, $\hat{h}_{mk} \rightarrow N \rightarrow 0$, $M \rightarrow \infty$ for $k \neq j$, which is equivalent to
$$\sum_{k=1}^{M} \sqrt{\frac{P_{\text{t},m}}{\alpha_{mk}^2}} \hat{h}_{mk} \rightarrow 0 \quad \text{for} \quad k \neq j,$$
where
- $\hat{h}_{mk} \sim \mathcal{C}\mathcal{N}(0, I_{N})$: small-scale fading coefficients.
- Alternative: consider the complementary CDF of the inner product of two given users’ channel
$$P_{b} = \Pr \left( \hat{h}_{mk} \geq \epsilon \right).$$

Objective: making $P_{b}$ very small to achieve Favorable orthogonality between users’ channel vectors, and therefore Favorable propagation.

Invoking Chebychev’s inequality, $P_{b}$ can be lower-bounded by
$$P_{b} = \Pr \left( \hat{h}_{mk} \geq \epsilon \right) \leq \frac{1}{\epsilon^2} \sum_{k=1}^{N} \alpha_{mk}^2 
\leq \frac{1}{\epsilon^2} \sum_{k=1}^{N} \alpha_{mk}^2,$$
which is design a scheme to minimize $P_{b}$.

4 Graphical Modeling and proposed solution

4.1 Scheduling design

1. Step 1: Construct a spatial correlation graph $G(V, E)$
2. Step 2: Group active users such that the spatial correlation between the channels is minimized.

4.2 Problem formulation and algorithm design

Define the following variable
$$x_{uk} = \begin{cases} 1 & \text{if user } u \text{ is allocated to the } k\text{-th group} \\ 0 & \text{otherwise} \end{cases}$$

The user grouping problem is formulated as
$$\max_{x} \sum_{u \in V} \sum_{k \in C} w_{uk} (1 - x_{uk}) \nu_{uk}$$
subject to
$$\sum_{k \in C} x_{uk} \leq 1, \quad \forall v \in V,$$
$$\sum_{k \in C} x_{uk} = 1, \quad \forall v \in V,$$
where:
- $C$: total number of groups.
- $\nu_{uk}$: maximum number of groups to which a user can belong at the same time.

Lemma 1: Computational tractability

Problem (1) is NP-hard in general.

GOAL: Design a low-complexity algorithm to sub-optimally solve problem (1). Define following variables and changes of variables
$$x_{uk} \triangleq (x_{uk,1}, \ldots, x_{uk,L})^{T}, y_{uk} \triangleq 2x_{uk} - 1, \quad w_{uk} \triangleq \left( \begin{array}{c} w_{uk,1} \\ \vdots \\ w_{uk,L} \end{array} \right),$$
where
- $1_L$: entry one column vector.

Combining SEMIDEFINITE RELAXATION method with the SCUBA COMPLEMENT, problem (1) can be relaxed as
$$\max_{y_{uk}} \sum_{k \in C} \left( \langle y_{uk}, (W_{uk}) \rangle \right)$$
subject to
$$\langle y_{uk}, 1 \rangle \leq 0, \quad \forall k \in C,$$
$$\langle y_{uk}, (W_{uk}) \rangle \geq 0, \quad \forall k \in C,$$
$$\langle y_{uk}, Y \rangle \leq \beta, \quad \forall \beta \in \mathbb{R}^+,$$
$$\langle y_{uk}, V \rangle \leq \theta, \quad \forall \theta \in \mathbb{R}^+,$$
where
- $\langle \cdot, \cdot \rangle$: inner product.
- $\beta$: downlink transmit power.
- $\theta$: set of users that belong to group $c$.

Problem (2) is a standard convex optimization problem and can be efficiently solved using CVX.

We develop a randomized procedure, in the vein of Gaussian randomization, to convert the optimal solution of (2) into a feasible solution to problem (1).

5 Numerical Results

Figure 3: Comparison of CDFs of normalized large-scale fading correlation for $K = 20$, $\alpha = 6$, $C = 4$.

Figure 4: Average Downlink Throughput versus the number of APs $M$.

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