We obtain an estimate $\hat{R}_d$ from $R^d$.

**Existing Approaches: Issues**

- Most of the available solutions rely on simple channel models and/or require specific array geometries (e.g., ULA).
- They often fail to address important effects such as propagation in 3D environments, polarization, and non-ideal array geometries.

**A Novel Covariance Model for Dual-polarized Arrays**

$$ R = E[hh^H] = \int_\Omega \rho_V(\theta)v(\theta)v(\theta)^H d\theta + \int_\Omega \rho_H(\theta)a(\theta)a(\theta)^H d\theta $$

- $\Omega = [-\pi, \pi] \times [0, \pi]$ is a spherical coordinate system.
- $v, a : \Omega \to \mathbb{C}^{N \times 1}$ are the BS antenna array responses for the vertical and for the horizontal polarizations.
- $\rho_V, \rho_H : \Omega \to \mathbb{R}^+$ are the frequency invariant angular power spectra for the vertical and for the horizontal polarizations (V-APS, H-APS).
- It can be derived from 3GPP-3D-like channel models, both for narrow-band and wideband OFDM systems.

**Extension of the ideas in [1] to the considered realistic covariance model.**

- Derived by focusing on the Hilbert space $\mathcal{H} = L^2[\Omega] \times L^2[\Omega]$ equipped with the inner product $\langle f, g \rangle = \int_\Omega f(\theta)g(\theta)d\theta$.
- We estimate $(\rho_V, \rho_H) \in \mathcal{H}$ by solving

$$ \text{find } (\hat{\rho}_V, \hat{\rho}_H) \in V : = r_{m=1}^M V_m, $$

where $V_m := \{ (\rho_V, \rho_H) \mid \langle g(m), \rho_V \rangle, \langle g(m), \rho_H \rangle \}$.

- $r_{m=1}^M V_m$ is the set of solutions of the problem $\{ R \} \implies \{ R\}$.
- Among the solutions, we choose the projection onto the linear variety $P(0)$ (Algorithm 1).
- Closed-form solution available in terms of inner products of the type $\langle g(m), \rho_V \rangle$.
- A more accurate but more complex variant (Algorithm 2) is available, which takes into account the positivity of the APS.

**Main Advantages**

- *Algorithm 1* is a simple matrix/vector multiplication $\phi^T F \phi$ over vectorized covariances.
- $F$ depends only on the array geometry and it is computed once for the entire system lifetime.
- It takes into account polarization and 3D propagation.
- No specific array geometry is required.