1. Continuous Phase Modulation

1.1. Motivation
- UAV (Unmanned Aerial Vehicle) control link by SatCom
- Robustness to non-linearity (due to its constant complex envelope)
- Usually used over AWGN channels
- No discrete equivalent channel in case of frequency-selective channels

1.2. CPM signals
\[ a_k = \alpha_k A_k \] sequence of \( N \) symbols taken in the M-ASK alphabet
Transmitted signal (complex envelop)
\[ s(t) = \exp(j2\pi f_d t + a(t) ) \]
where \( a(t) \) is the information phase.

PAM Decomposition
Laurent representation [1] (extended by Mengali and Morelli to the M-ary case)

2. A linear representation

2.1. Equivalent channels

2.1.1. Received Signal

Received signal
\[ r(t) = \Gamma(f_d) A h(t) + w(t) \]
with \( w(t) \) complex Gaussian white noise with spectral density \( N_0 \)

2.2. Matrix-wise representation

Let define \( L \) the equivalent channel length and for \( k = 0 \) to \( K - 1 \):

\[ A_k = \begin{pmatrix} a_k & 0 & \cdots & 0 \\ a_{k+1} & a_k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{K-1} & \cdots & a_{K-2} & a_{K-1} \end{pmatrix} \]

\[ \Gamma(f_d) = \exp \left( j2\pi f_d L T \right) \]
and \( h_{k,i} = [h_{k,i}] \)\]

\[ A = [A_0, A_1, \ldots, A_{K-1}] \]

\[ h_{k,i} = [h_{k,i} A_{k,i}] \]

Matrix model of the received signal
\[ r = \Gamma(f_d) A h_q + w \]

2.3. Joint ML Estimation

Likelihood function (similar to [1] for linear modulations and binary GMSK)
\[ \Delta r \Rightarrow \hat{f}_d = \frac{1}{\sigma_r^2} \exp \left\{ -\frac{1}{2} \Gamma(f_d) A h_q [r - \Gamma(f_d) A h_q] \right\} \]

Least Squares Estimation of the equivalent channels for a fixed \( \hat{f}_d \)
\[ \hat{h}_{eq,k} = (A^H A)^{-1} A^H r \]

 Likelihood function (to maximize) for Carrier Frequency Estimation using Eq (13) in Eq (12)
\[ \hat{g}'(\hat{f}_d) = -\rho(\hat{f}_d) + 2\pi \sum_{m=0}^{L-1} \sum_{k=0}^{L-1} a_k a_{k+m} e^{-j2\pi m\hat{f}_d} \]

Hence,
\[ \hat{f}_d = \arg \max \hat{g}'(\hat{f}_d) \]

Procedure
- Compute \( \hat{f}_d \) using Eq (16)
- Counter-rotate the received signal \( r \) according to \( \hat{f}_d \)
- Compute \( \hat{h}_{eq} \) using Eq (13)

3. Results

Preamble of 128 symbols modulated by a GMSK with \( M = 4 \), the modulation index \( h = 1/4 \), the CPM memory \( L_{cpm} = 3 \) and \( BT = 0.3 \), and a FFT of size 4096.
Uniformly random CFO between \(-0.5R_s\) and \(0.5R_s\).
We consider the urban GSM channel and that all equivalent discrete channels have a length \( K = 8 \).

4. References


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