A Joint Perspective of Periodically Excited Efficient NLMS Algorithm and Inverse Cyclic Convolution

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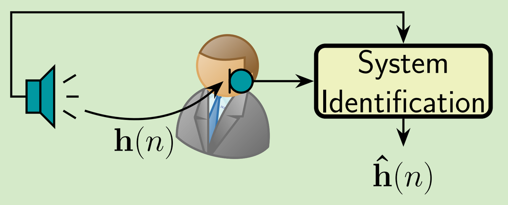
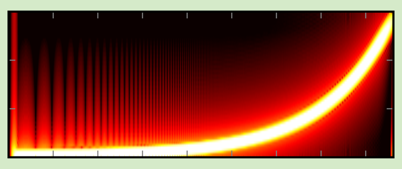
1 Introduction: System Identification



Example: Fast measurements of HRTFs
→ Different approaches exist

For static acoustic measurements: *Inverse Cyclic Convolution* (ICC) with *Exponential Sweeps*



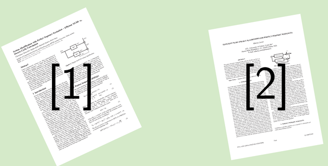
→ ICC

For dynamic tracking of systems: *Normalized Least Mean Square* (NLMS)-type algorithms with so-called *Periodic Perfect Sequences* (PPSEQ)

$$\varphi_{xx}(\lambda) = \begin{cases} E_x & \text{for } \lambda \bmod N = 0 \\ 0 & \text{otherwise} \end{cases}$$ PPSEQ

→ NLMS

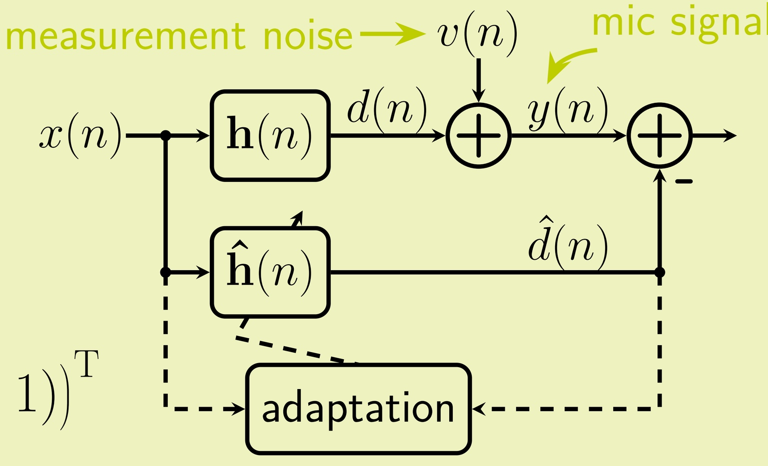
Equivalence of ICC and NLMS for PPSEQ input [1], based on efficient NLMS implementation [2]



This work is a generalization of [1] showing the equivalence also for *arbitrary* periodic input signals for the NLMS implementation in [3]

2 System Model

- System to be identified:
  $\mathbf{h}(n) = \big(h_0(n)\ h_1(n) \cdots h_{N-1}(n)\big)^{\mathrm{T}}$
- Excitation vector:
  $\mathbf{x}(n) = \big(x(n)\ x(n-1) \cdots x(n-N+1)\big)^{\mathrm{T}}$
- Output of system:
  $d(n) = \mathbf{h}^{\mathrm{T}}(n) \cdot \mathbf{x}(n)$



correspondingly: $\hat{\mathbf{h}}(n)$, $\mathbf{d}(n)$, $\hat{\mathbf{d}}(n)$, $\mathbf{y}(n)$

i Notation & Auxiliary Operators

DFT matrix F

Discrete Fourier Transform via DFT matrix $\mathbf{F}$

DFT: $\mathbf{X}(n) = \mathbf{F} \cdot \mathbf{x}(n)$

IDFT: $\mathbf{x}(n) = \mathbf{F}^{-1} \cdot \mathbf{X}(n)$

$$\mathbf{F} = \begin{pmatrix} \omega^{0\cdot 0} & \omega^{0\cdot 1} & \cdots & \omega^{0\cdot(N-1)} \\ \omega^{1\cdot 0} & \omega^{1\cdot 1} & \cdots & \omega^{1\cdot(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^{(N-1)\cdot 0} & \omega^{(N-1)\cdot 1} & \cdots & \omega^{(N-1)\cdot(N-1)} \end{pmatrix}$$

with $\omega = e^{-j\frac{2\pi}{N}}$

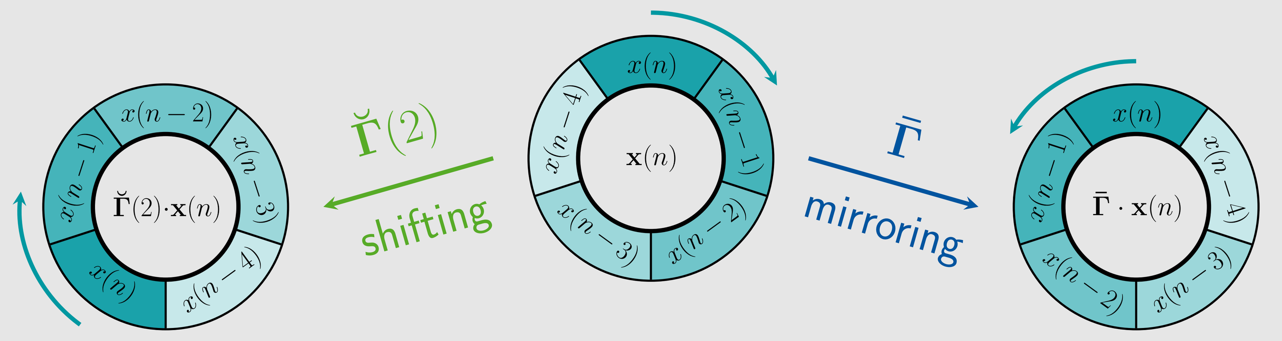
Permutation matrix Γ

Permutation matrix $\boldsymbol{\Gamma}(n)$ reorders elements of vector, beginning with n-th element going cyclically backwards through vector

Example: $N = 5,\ n = 2$

$$\boldsymbol{\Gamma}(n) = \begin{pmatrix} 0&0&1&0&0 \\ 0&1&0&0&0 \\ 1&0&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&1&0 \end{pmatrix} = \begin{pmatrix} 0&0&0&1&0 \\ 0&0&0&0&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \end{pmatrix} \cdot \begin{pmatrix} 1&0&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&1&0 \\ 0&0&1&0&0 \\ 0&1&0&0&0 \end{pmatrix}$$

shifting matrix $\breve{\boldsymbol{\Gamma}}(n)$ — mirroring matrix $\bar{\boldsymbol{\Gamma}}$



3 Efficient NLMS Algorithm (eNLMS)[3]

NLMS

Novel description of [3]

$$\hat{\mathbf{h}}_{\mathrm{eNLMS}}(n) = \mathbf{F}^{-1}\mathrm{diag}\{\mathbf{F}\mathbf{w}_0\}\mathbf{F}\hat{\mathbf{c}}(n)$$ ← see i

Use NLMS algorithm for transformed coefficients:

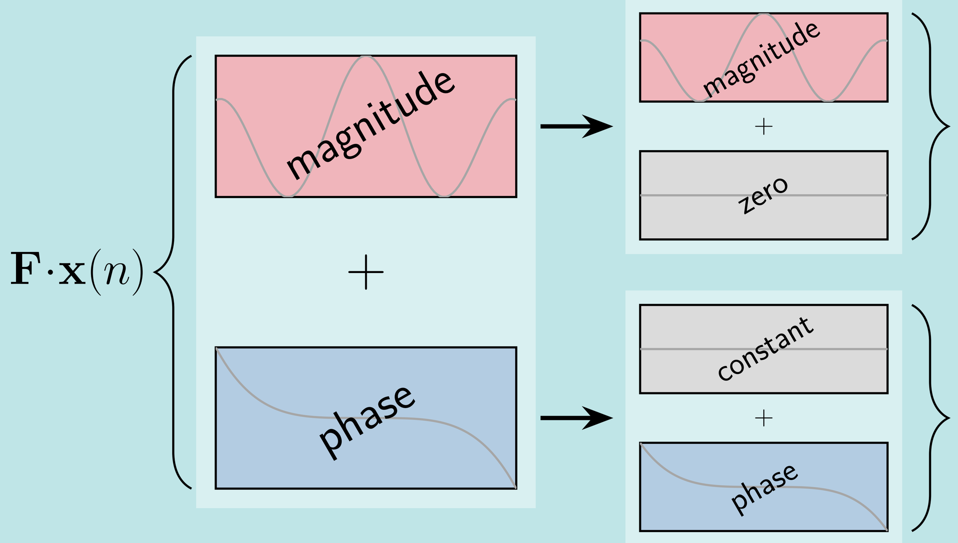
$$\hat{\mathbf{c}}(n+1) = \hat{\mathbf{c}}(n) + \mu\big(y(n) - \hat{\mathbf{c}}^{\mathrm{T}}(n)\cdot\mathbf{e}_{n \bmod N}\big)\mathbf{e}_{n \bmod N}$$

unit vector $\mathbf{e}_i = (0 \cdots 1 \cdots 0)^{\mathrm{T}}$ (1 at position i, indices $0 \ldots N-1$)

step-size parameter μ

Alternative System Description

Idea: Transform $\mathbf{h}(n)$ (basis $\mathbf{e}_i$) to $\mathbf{c}(n)$ (basis $\mathbf{w}_i$) with respect to $\mathbf{x}(n)$



$\mathbf{F}\cdot\mathbf{s}$: zero-phase component

$\mathbf{x}(n) = \mathbf{F}^{-1}\mathrm{diag}\{\mathbf{F}\mathbf{s}\}\mathbf{F}\mathbf{p}(n)$

$\mathbf{F}\cdot\mathbf{p}(n)$: const.-magn. component
↳ $\mathbf{x}(n)$ periodic in N
→ only N mutually orthogonal $\mathbf{p}_i$, $i = 0,\ldots,N-1$ exist.
with $\mathbf{p}_{n \bmod N} = \mathbf{p}(n) = \breve{\boldsymbol{\Gamma}}(n)\mathbf{p}_0$
Energy: $E_p = \mathbf{p}_0^{\mathrm{T}}\mathbf{p}_0$

- Goal: $\mathbf{w}_i^{\mathrm{T}}\mathbf{x}(n) = \begin{cases} 1 & \text{for } i = n \bmod N \\ 0 & \text{otherwise} \end{cases}$
- To obtain basis $\mathbf{w}_i$:
  $$\mathbf{w}_i = \tfrac{1}{E_p}\mathbf{F}^{-1}\mathrm{diag}\{\mathbf{F}\mathbf{s}\}^{-1}\mathbf{F}\mathbf{p}_i$$
  normalization to E_p — equalization with $\mathbf{s}$ — 'direction' of $\mathbf{p}_i$

  → $\mathbf{w}_i$ form non-orthogonal basis of $\mathbb{R}^N$
- Output of system can be expressed as:
  $$d(n) = \sum_{i=0}^{N-1} h_i(n)\mathbf{e}_i^{\mathrm{T}}\mathbf{x}(n),\quad h_i \text{ coeff. to basis } \mathbf{e}_i$$
  $$= \sum_{i=0}^{N-1} c_i(n)\mathbf{w}_i^{\mathrm{T}}\mathbf{x}(n),\quad c_i \text{ coeff. to basis } \mathbf{w}_i$$
  change of basis

  → $\mathbf{x}(n)$ appears as unit impulse excitation in transform domain

  Current output sample matches exactly one coefficient:
  $d(n) = c_{n \bmod N}(n)$ respectively $\mathbf{d}(n) = \boldsymbol{\Gamma}(n)\mathbf{c}(n)$
- $\mathbf{h}(n) = \sum_{i=0}^{N-1} c_i(n)\mathbf{w}_i = \cdots = \mathbf{F}^{-1}\mathrm{diag}\{\mathbf{F}\mathbf{w}_0\}\mathbf{F}\boldsymbol{\Gamma}(n)\mathbf{d}(n)$

4 Inverse Cyclic Convolution (ICC)

Deconvolution in frequency domain by element-wise division:

$$\hat{\mathbf{h}}_{\mathrm{ICC}}(n) = \mathbf{F}^{-1}\big[\mathrm{diag}\{\mathbf{F}\mathbf{x}(n)\}^{-1}\mathbf{F}\mathbf{y}(n)\big]^{*}$$

← required to compensate for mirroring of sequences $\mathbf{x}(n)$ and $\mathbf{y}(n)$

5 Equivalence of eNLMS and ICC

$\mu = 1$

By alternative system description in 3 and 4, equivalence for $\mu = 1$ can be shown:

$$\hat{\mathbf{h}}^{\mu=1}_{\mathrm{eNLMS}}(n) \overset{!}{=} \hat{\mathbf{h}}_{\mathrm{ICC}}(n)$$ → see paper

μ arbitrary

For arbitrary step-sizes μ, eNLMS yields:

$$\hat{\mathbf{c}}(n+N) = \hat{\mathbf{c}}(n) + \mu\big(\boldsymbol{\Gamma}(n+N-1)\mathbf{y}(n+N-1) - \hat{\mathbf{c}}(n)\big)$$
$$= (1-\mu)\cdot\hat{\mathbf{c}}(n) + \mu\cdot\big(\boldsymbol{\Gamma}(n+N-1)\mathbf{y}(n+N-1)\big)$$

$$\hat{\mathbf{h}}_{\mathrm{eNLMS}}(n+N) = (1-\mu)\cdot\hat{\mathbf{h}}_{\mathrm{eNLMS}}(n) + \mu\cdot\hat{\mathbf{h}}^{\mu=1}_{\mathrm{eNLMS}}(n+N)$$

As $\hat{\mathbf{h}}^{\mu=1}_{\mathrm{eNLMS}}(n) = \hat{\mathbf{h}}_{\mathrm{ICC}}(n)$ → recursive averaging can be applied to ICC

Complexity is identical (see paper for detailed description)

6 Conclusion

- ICC and eNLMS are mathematical identical
- Bridged gap between ICC for static acoustic measurements and the eNLMS algorithm for dynamic system identification and tracking

▶ Enables transfer of knowledge from both approaches:
- from ICC to eNLMS: regularization methods
- from eNLMS to ICC: dynamic tracking properties and step-size control

7 References

[1] C. Antweiler, S. Kühl, B. Sauert, and P. Vary, "System Identification with Perfect Sequence Excitation – Efficient NLMS vs. Inverse Cyclic Convolution", in ITG Fachtagung Speech Communication, Erlangen, Germany, Sept. 2014.

[2] A. Carini, "Efficient NLMS and RLS Algorithms for Perfect Periodic Sequences," in Proc. of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Dallas, TX, USA, Mar. 2010, pp. 3746–3749.

[3] A. Carini, "Efficient NLMS and RLS Algorithms for Perfect and Imperfect Periodic Sequences", IEEE Transactions on Signal Processing, vol. 58, no. 4, pp. 2048-2059, Apr. 2010.