Fast Double-coupled Nonnegative Canonical Polyadic Decomposition
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Introduction

background
• Coupled tensor decomposition has become a popular technique for the simultaneous analysis of multiblock tensors [1].
• It enables the simultaneous extraction of common components and individual components.
• It is reasonable to expect identical elicited information among subjects since ongoing EEG are collected under the same stimulus.
• Time consumption would go extremely heavy due to the high-dimensional and non-negative nature of ongoing EEG.

Objective
To develop an efficient data-driven coupled tensor decomposition algorithm.

Proposed algorithm

Coupled tensor decomposition (or LCPTD [2]) model
• Each factor matrix \( U^{(n,s)} = [U_i^{(n,s)}, U_j^{(n,s)}] \) consists of two parts: \( U_i^{(n,s)} \in \mathbb{R}^{I_i \times L_i L_s} \), \( 0 \leq L_i \leq R \) shared by all tensors with coupling information and \( U_j^{(n,s)} \in \mathbb{R}^{I_j \times (R-L_i L_s)} \) representing individual characteristics of each single tensor block.

\[ \begin{align*}
    X^{(n,s)} = U_i^{(n,s)} C_i + U_j^{(n,s)} C_j + \nu^{(n,s)},
\end{align*} \]

Fig 1. Conceptual illustration of dual-coupled LCPTD model

Realization of FDC-NCPD
• Squared Euclidean Divergence minimization
• Hierarchical Alternating Least Squares (HALS)
• Fast Hierarchical Alternating Least Squares (Fast HALS [3])
• The object function can be expressed as:

\[ \begin{align*}
    & \text{minimize} \quad \sum_{s=1}^{S} \left[ X^{(s)} - \sum_{r=1}^{R} \lambda_r^{(s)} u_r^{(1,s)} \circ u_r^{(2,s)} \circ \cdots \circ u_r^{(N,s)} \right]_F^2, \\
    & \text{s.t.} \quad u_r^{(n,s)} = \cdots = u_r^{(n,S)} \quad \forall n, r \leq L_n, \\
    & \quad \| u_r^{(n,s)} \|_F = 1, \quad n = 1 \ldots N, \quad r = 1 \ldots R, \quad s = 1 \ldots S
\end{align*} \]

Experiments and Results

Exp1. Validation of synthetic data
• NTF-HALS, NTF-FastHALS, LCPTD-HALS and FDC-NCPD
• Convergence speed: Execution time and iteration number, 30 runs
\( SNR = 20 \text{ dB}, L_{1,2,3} = (7n, 8n, 9n), R = 4n, L_{1,2} = 2n, S = 10 \)
• Decomposition quality: Fit and PI, 20 runs
\( SNR = 5 \sim 20 \text{ dB}, L_{1,2,3} = (40, 50, 60), R = 30, L_{1,2} = 20, S = 10 \)
• Evaluation index: Execution time, iteration number, Fit and PI

Exp2. Application of multi-subject ongoing EEG data
• Data collection, data preprocessing can be found in [4]
• Tensor (14): 16 channels \( \times 146 \) frequency bins \( \times 510 \) samples
• The coupled information exists on the first two modes.
• DIFFIT suggested \( R = 36, L_{1,2} = 20 \)

<table>
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Fig 2. Results of Synthetic data

Fig 3. An example of temporal component with its corresponding spatial and spectral components in Exp2.

Conclusion and Future work

• Double coupled tensor-based using LCPTD model and Fast-HALS strategy greatly reduces the computational complexity without compromising the decomposition quality.
• Further analyze brain activation regions and frequency oscillations corresponding to the significantly correlated temporal components

References

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