Region-to-region kernel interpolation of acoustic transfer function with directional weighting

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**Background**
- The environment effects signal recordings. ⇒ Reflections, diffraction and other physical phenomena.
- We calculate the acoustic transfer function (ATF) to represent these effects.
- We wish to represent the ATF in a region-to-region basis, using ATF measurements alone.
- The ATF can be represented using sound field analysis techniques.

**Problem statement**
- The ATF can be divided into a direct component $h_D(r|s, k)$ and a reverberant component $h_R(r|s, k)$.
- $h_D(r|s, k) = \frac{e^{ik|r-s|}}{4\pi |r-s|}$ Assumed to be point source in the free-field.
- The reverberant component can be approximated using kernel ridge regression. ⇒ We have shown this method performs better than the wavefunction expansion in [Ribeiro+, 2020].
- The interpolation function will be:

$$\tilde{h}_R(r|s) = \kappa(r|s)(K + \lambda I)^{-1}y$$

where $\kappa(r|s) = [\kappa(r|s, q_1), \ldots, \kappa(r|s, q_N)]$ and

$$K = \begin{bmatrix}
\kappa(q_1, q_1) & \kappa(q_1, q_2) & \cdots & \kappa(q_1, q_N) \\
\vdots & \ddots & \ddots & \vdots \\
\kappa(q_N, q_1) & \kappa(q_N, q_2) & \cdots & \kappa(q_N, q_N)
\end{bmatrix}$$

But what kernel function should we use?

⇒ Embedding directionality into the kernel has been shown to improve sound field interpolation results in [Ito+, 2020].

**Proposed method**
- We employed the Herglotz wavefunction:

$$h_R(r|s) = \mathcal{I} \hat{h}_R(r|s) ,$$

$$\mathcal{I}(f; r|s) := \int_{S^2 \times S^2} e^{ik(f(r+s) + \hat{r})} f(r, \hat{r}) d\hat{r} d\hat{s}$$

Superposition of plane waves

- Which allowed us to define the following function set and inner product, forming a Hilbert space:

$$\mathcal{H} = \{ h_R = \mathcal{I} \hat{h}_R(r|s) : \hat{h}_R \in L^2(W, S^2 \times S^2), \hat{h}_R(r, \hat{s}) = \hat{h}_R(\hat{s}, \hat{r}) \forall \hat{r}, \hat{s} \in S^2 \}$$

$$\langle f, g \rangle_{\mathcal{H}} = \int_{S^2 \times S^2} \frac{f(r, \hat{s}) \overline{g(\hat{r}, \hat{s})}}{W(\hat{r}, \hat{s})} d\hat{r} d\hat{s} \forall f, g \in \mathcal{H}$$

Informing this kernel function:

$$\kappa(r|s, r'|s') = \mathcal{I} \left( W(\hat{r}, \hat{s}) \left( e^{-ik(r-r')} + e^{-ik(r'-r'' + r')} \right) ; r|s \right)$$

⇒ Making the space a reproducing kernel Hilbert space(RKHS).
- We chose the following weight function:

$$W(\hat{r}, \hat{s}) = w(\hat{r}) \psi(\hat{s})$$

$$w(y) = \frac{1}{4\pi} \left( 1 + \gamma^2 - \frac{\cosh(\beta y - \nu_0)}{\cosh(\beta)} \right)$$

Favors early reflections

- Considering the direct component is removed, the weight was made minimal in the direction connecting the centers.
⇒ The hyperparameter $\beta$ controls the width of the cavity, $\gamma$ the depth.
- Parameters were obtained by optimizing leave-one-out (LOO) cross-validation of the square error(SQE) and Tukey biweight loss.

$$\text{SQE}(\sigma) = |z|^2$$

$$\text{Tukey}(\sigma) = \left( \frac{\sigma^2}{6} - \frac{1}{2} \left( 1 - \left( \frac{|z|}{\sigma} \right)^2 \right)^3 \right) \cdot |z| \leq \sigma$$

Robust loss function

**Experimental results**
- Simulations with the image source method.
  - Room dimensions: 3.2 m × 4.0 m × 2.7 m.
  - Reverberation time: $T_K = 0.45$ s.
  - Radius of both regions: 0.2 m.
  - Centers of $\Omega_N$: $\pm(0.35, 0.43, 0.29)$ m.
- We compared the proposed directionally-weighted kernels to the uniform kernel derived when $\nu = (4\pi)^{-1}$.

Normalized mean square error (NMSE)

**Reconstruction of impulse response (950Hz)**

- Original
- Uniform
- SQE
- Tukey

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Experimental results</th>
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<tbody>
<tr>
<td>200</td>
<td>-20</td>
</tr>
<tr>
<td>500</td>
<td>-15</td>
</tr>
<tr>
<td>1000</td>
<td>-10</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares (dB)</th>
<th>Sources</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
<td>-5</td>
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<tr>
<td>-10</td>
<td>SQE</td>
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<td>-15</td>
<td>Tukey</td>
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