Millimeter Wave MIMO Channel Estimation with 1-bit Spatial Sigma-delta Analog-to-Digital Converters

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Millimeter-wave MIMO communication

- mmWave MIMO systems can provide cellular data rates of the order of **Giga bit per second**.

- Dedicated high-resolution quantizer per radio frequency (RF) chain is expensive.

- Solution: Use low-resolution/coarse data converters. Issue: degradation in performance!
Sigma-delta quantization

➢ Classical technique to increase effective resolution of quantizers.

\[ Q(x) = b \text{sign}(\Re(x)) + j \text{sign}(\Im(x)) \]

\[ r[n] = x[n] + r[n-1] - y[n-1] \]

\[ y[n] = Q(r[n]) \]

\[ e[n] = r[n] - y[n] \]

\[ R(z) = X(z) + z^{-1}E(z) \]

\[ Y(z) = R(z) - E(z) \]

\[ Y(z) = X(z) - (1 - z^{-1})E(z) \]

➢ Shape quantization noise to higher frequencies ➢ Higher effective resolution for low-pass signals.

➢ Expensive when \( f_{Nyq} \) is high.
Spatial sigma-delta quantization

- Spatial oversampling followed by spatial feedback.

- Noise is shaped towards higher spatial frequencies!

- Higher effective resolution for spatial low-pass signals.

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 1 & 1 & \cdots & \cdots & 1 \end{bmatrix}, \quad V = U - I_N \]

\[ y = Q(Ux - Vy) \]
We need to estimate \( \{\alpha, \theta, \phi\} \) using the knowledge of \( P \) and \( T \).
Prior art

- LMMSE channel estimator using Bussgang Decomposition and elementwise Bussgang Decomposition. [S. Rao, et al. 2020]


Limitations in prior art

- Needs the knowledge of **channel correlation** to compute the Bussgang decomposition \((C_x, C_{yx})\) and to set voltage levels.

- In mmWave, **knowing channel correlation** amounts to **knowing angles**. Not practical.

- Existing method directly estimates \(\mathbf{H}\) instead of estimating \(\{\alpha, \theta, \phi\}\). Does not exploit parametric channel model.

\[
N_t N_r \quad \text{unknowns} \\
3 \quad \text{unknowns}
\]

\[
N_t N_r \gg 3
\]
Quantization noise modeling in sigma-delta quantizer

➢ Usual method of noise modelling: \( Q(r_i) = r_i + e_i \)

Property - I:
When the dynamic range of \( r_i \) is large and \( Q \) is a multi-level quantizer with large number of levels, we can assume that \( e_i \) is uniformly distributed and uncorrelated with \( r_i \).

➢ Not reasonable to assume that \( r_i \) and \( e_i \) are uncorrelated for one-bit quantizers.

➢ Closed form expressions relating \( e_i \) to \( r_i \). [R. M. Gray, et. al. 1989]

➢ We propose a new noise-model for spatial sigma-delta ADC.

Quantization noise modeling in sigma-delta quantizer

➢ Linearize the quantizer: \( y_i = Q(r_i) = r_i + e_i \) → No assumptions about the pdf !!

➢ We can show that

\[
\Re(e_i) = b - (2b) \left( \frac{(i-1)}{2} + \sum_{k=1}^{i} \frac{\Re(x_k)}{2b} \right), \quad \forall \ i = 1, 2, \ldots, N_r
\]

\[
\langle x \rangle = x - \text{floor}(x), \quad \forall \ x \in \mathbb{R}
\]

\[
\frac{1}{2b} \mathbf{U} \Re(y) + \frac{1}{2} \mathbf{V} \mathbf{1} - \frac{1}{2} \mathbf{1} = \text{floor}(\frac{1}{2b} \mathbf{U} \Re(x) + \frac{1}{2} \mathbf{V} \mathbf{1})
\]

\[
\frac{1}{2b} \mathbf{U} \Im(y) + \frac{1}{2} \mathbf{V} \mathbf{1} - \frac{1}{2} \mathbf{1} = \text{floor}(\frac{1}{2b} \mathbf{U} \Im(x) + \frac{1}{2} \mathbf{V} \mathbf{1})
\]
Quantization noise modeling in sigma-delta quantizer

- **Linearize the floor function**
  \[
  \text{floor}\left(\frac{1}{2b}U\mathbb{R}(x) + \frac{1}{2}V1\right) = \frac{1}{2b}U\mathbb{R}(x) + \frac{1}{2}V1 + \mathbb{R}(q)
  \]
  \[\mathbb{R}(q_i) \in (-1, 0] \forall i = 1, \ldots, N_t\]

- **Dynamic range** of \(\left[\frac{1}{2b}U\mathbb{R}(x) + \frac{1}{2}V1\right]_i\) can be large for large \(i\)

- **floor(.)** is like a multi-level quantizer

- **Using Property-1**, we have

For large antenna indices, \(q_i\) is uniformly distributed and is uncorrelated with \(\left[\frac{1}{2b}Ux + \frac{1}{2}V1\right]_i\)
Quantization noise modeling in sigma-delta quantizer

➢ Re-arranging

\[
y = x + (2b)U^{-1}\tilde{q},
\]
\[
\tilde{q} = (\Re(q) + \frac{1}{2}I) + j(\Im(q) + \frac{1}{2}I).
\]

➢ In practice, we can consider **Property-2** to be satisfied by all antennas in massive MIMO systems.

➢ Thus, \(\tilde{q}\) is uncorrelated to \(x\) and

\[
\mathbb{E}[\tilde{q}\tilde{q}^H] = \frac{1}{6}I
\]

\[
\Re(\tilde{q}_i), \Im(\tilde{q}_i) \sim \text{Unif}(-0.5, 0.5) \text{ and i.i.d}
\]

Property – 2:
For large antenna indices, \(q_i\) is uniformly distributed and is independent with \(\left[\frac{1}{2b}Ux + \frac{1}{2}V1\right]_i\)
Pilot selection at the MS

- We desire $S \in \{-1, 1\}^{N_t \times M}$ to be an orthogonal matrix with $SS^H = S^H S = 2N_t I_{N_t}$

- Let $S = G + jG$ → Hadamard matrix

- Choose $T$ to obtain desired $S$

- We have

$$\frac{1}{2} U G + \frac{1}{2} V 1\!\!1^T - \frac{1}{2} 1\!1^T = \text{floor}(\frac{1}{2} U \Re(T) + \frac{1}{2} V 1\!\!1^T)$$

$$\frac{1}{2} U G + \frac{1}{2} V 1\!\!1^T - \frac{1}{2} 1\!1^T = \text{floor}(\frac{1}{2} U \Im(T) + \frac{1}{2} V 1\!\!1^T)$$

- Many ways to choose $T$. By noting $\text{floor}(x) = x, \forall x \in \mathbb{Z}$, we may choose

$$T = (G - U^{-1} 1\!\!1^T) + j(G - U^{-1} 1\!\!1^T)$$
Channel estimation at the BS

- Received signal at the BS

\[ Y = \sqrt{\frac{P}{2N_t}} HS + N \rightarrow \text{Gaussian + quantization noise} \]

- Noise covariance matrix

\[ R_n = I_{N_x} + \frac{2b^2}{3} U^{-1} U^{-H} \rightarrow \text{Not spatially white!} \]

- To estimate angles, we can use subspace based methods, e.g., MUSIC.

\[ \hat{H} = R_n^{-1/2} Y \frac{1}{\sqrt{2PN_t}} S^H = R_n^{-1/2} H + R_n^{-1/2} NSH / \sqrt{2PN_t} \]

Pre-whitening matrix

\[ \hat{H} \]

Whitened noise term \( \tilde{N} \)

- We have

\[ \hat{H} = R_n^{-1/2} \alpha_{BS}(\theta) a_{UE}(\phi) + \tilde{N} \]

Signal part

Noise part (white and uncorrelated with signal part)
Channel estimation at the BS

- The SVD of $\hat{H}$ is given by $\hat{H} = P \Sigma Q^H$

- $P = [p_s \quad P_n]$, $Q = [q_s \quad Q_n]$

- $P \in \mathbb{C}^{N_r \times 1}$, $P_n \in \mathbb{C}^{N_r \times (N_r-1)}$, $Q_n \in \mathbb{C}^{N_t \times 1}$

- $\mathcal{R}(p_s) = \mathcal{R}(\hat{H}) = \mathcal{R}(R_n^{-1/2}a_{BS}(\theta))$, $\mathcal{R}(q_s) = \mathcal{R}(\hat{H}^H) = \mathcal{R}(a_{MS}(\phi))$

- AoA and AoD can be estimated by finding peaks of MUSIC pseudo-spectra

$$\rho_{BS}(\tilde{\theta}) = \frac{1}{\|P_n^H R_n^{-1/2}a_{BS}(\tilde{\theta})\|_2^2}; \quad \rho_{MS}(\tilde{\phi}) = \frac{1}{\|Q_n^H a_{MS}(\tilde{\phi})\|_2^2}$$

- Path gain $\alpha$ can be estimated using least squares.

$$\hat{\alpha} = d^H \hat{h} / \|d\|^2_2, \text{ where } d = \text{vec}(R_n^{-1/2}a_{BS}(\hat{\theta})a_{MS}^H(\hat{\phi})), \text{ and } \hat{h} = \text{vec}(\alpha R_n^{-1/2}a_{BS}(\hat{\theta})a_{MS}^H(\hat{\phi}) + \tilde{N})$$
Voltage level selection at the BS

- Voltage level selection is crucial to ensure good performance.

- Voltage level \( b \) needs to be selected to satisfy
  - \( \mathcal{L}(x_i) \approx x_i \), i.e., error due to clipping is minimum – Choose large level
  - \( \left[ \frac{1}{2b} \mathbf{U} \mathbf{x} + \frac{1}{2} \mathbf{V} \mathbf{1} \right]_i \), and \( q_i \) are uncorrelated – Choose small level

- Optimal selection is difficult.

- We have \( x_i \sim \mathcal{CN}(\mu, 1) \), \( \mu = \sqrt{\frac{P}{2N_t}} \alpha e^{2\pi d(i-1) \sin(\theta) / \lambda} a^H_M \phi \mathbf{s} \)

- \( |\mu| \leq \sqrt{2PN_t} \), if \( |\alpha| = 1 \)

- Choose \( b = \sqrt{2PN_t} + 3\sqrt{0.5} \) Variance of \( \Re(x_j), \Im(x_j) \)

- Choice works well for LoS channels.
Simulations

- \( N_t = 8, \ N_r = 128, \ d = \frac{\lambda}{8}, \ \theta, \ \phi \in [-\Theta^0, \Theta^0], |\alpha| = 1 \)

\[ \theta = \phi = 0^0, \ \text{SNR} = -5\text{dB} \]

Correlation between input and quantization noise

Channel estimation error

We have proposed angular channel estimation algorithm for LoS mmWave SU-MIMO systems with one-bit spatial ∑Δ quantizers.

Usual i.i.d. noise assumption not suitable for one-bit quantizers.

Presented a new noise modeling for spatial ∑Δ quantizers by linearizing the \text{floor}(.) function.

Input and quantization noise are uncorrelated for antennas away from phase reference.

Proposed algorithm does not require prior knowledge of channel correlation.

Significantly outperforms conventional one-bit MIMO systems, comparable to unquantized systems.
Thank you!

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