Sparse Tensor Recovery via N-mode FISTA with Support Augmentation[1]

Ashley Prater-Bennette
Air Force Research Laboratory
Rome, NY USA
ashley.prater-bennette@us.af.mil

Lixin Shen
Syracuse University
Syracuse, NY USA
lshen03@syr.edu

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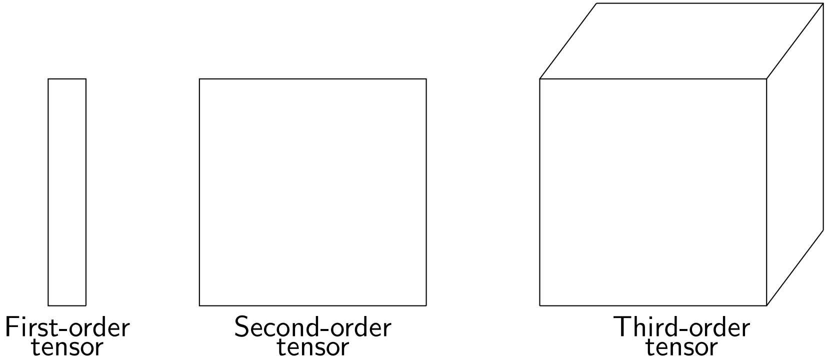
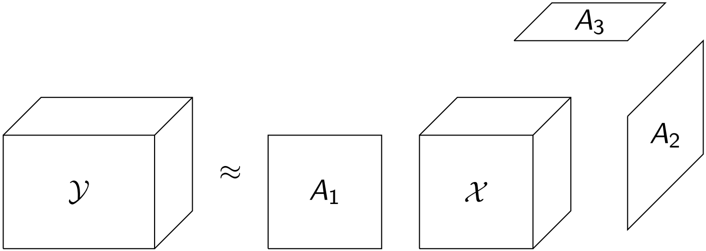
Tensors

Tensors are higher dimensional analogues of vectors.



Tucker Decomposition

$$\mathcal{Y} = \mathcal{X} \times_1 A_1 \times_2 A_2 \times_3 A_3,$$

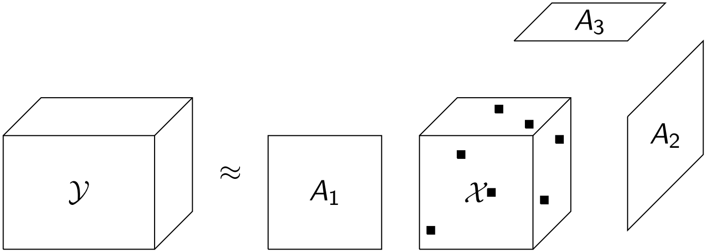


Sparse Tucker Decomposition

Goal:

Given $\mathcal{Y}$ and the collection $\{A_n\}$, find sparse $\mathcal{X}$ such

$$\mathcal{Y} \approx \mathcal{X} \times_1 A_1 \times_2 A_2 \times \cdots_N A_N + \varepsilon.$$



$$\underset{\mathcal{X}}{\operatorname{argmin}} \left\{ \|\mathcal{X}\|_1 + \frac{\lambda}{2} \|\mathcal{Y} - \mathcal{X} \times_1 A_1 \times_2 A_2 \cdots \times_N A_N\|_F^2 \right\}$$

Sparse Tucker Decomposition - Kronecker Approach

The Sparse Tucker problem may be recast as

$$\mathcal{X}^*_{\text{vec}} \approx \text{argmin} \left\{ \|\mathcal{X}_{\text{vec}}\|_1 + \frac{\lambda}{2} \|\mathcal{Y}_{\text{vec}} - P\mathcal{X}_{\text{vec}}\|_2^2 \right\},$$

where $P = A_N \otimes A_{N-1} \otimes \cdots \otimes A_1$,
and use compressed sensing-style techniques[2].

[2] [Dasarathy et.al. IEEE Trans Info Th. 2015], [Duarte, Baraniuk ICASSP 2010],[Caiafa, Cichocki ICASSP 2012]

Soft-thresholding Approach [3]

Algorithm 1 3-mode FISTA

1: **Inputs:** $\mathcal{Y},\ A_1, A_2, A_3,\ \lambda, \mathrm{tol}, L$

2: **Initialize:** $d_1 = 1,\ t = 1,\ \mathcal{X}_0 = \mathcal{Z}_1 = \mathcal{Y} \times_1 A_1^\top \times_2 A_2^\top \times_3 A_3^\top$

3: **while** (stopping criteria not met) **do**

4: $\quad \mathcal{Y}_t \leftarrow \mathcal{Z}_t \times_1 A_1 \times_2 A_2 \times_3 A_3$

5: $\quad \mathcal{X}_t \leftarrow \mathrm{prox}_{\frac{1}{\lambda L}\|\cdot\|_1} \left(\mathcal{Z}_t - \frac{1}{L} (\mathcal{Y}_t - \mathcal{Y}) \times_1 A_1^\top \times_2 A_2^\top \times_3 A_3^\top \right)$

6: $\quad d_{t+1} \leftarrow \frac{1+\sqrt{1+4d_t^2}}{2}$

7: $\quad \mathcal{Z}_{t+1} \leftarrow \mathcal{X}_t + \frac{d_t - 1}{d_{t+1}} (\mathcal{X}_t - \mathcal{X}_{t-1})$

8: $\quad t \leftarrow t + 1$

9: **end while**

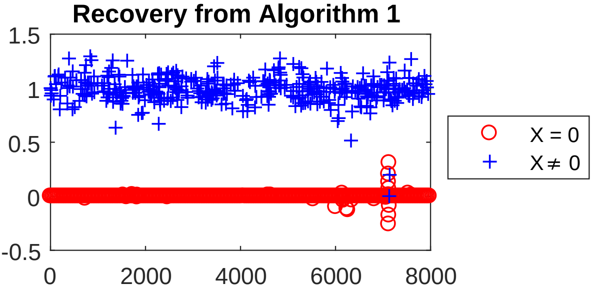
10: **Output:** $\mathcal{X}^{\mathrm{FISTA}}$ final $\mathcal{X}_t$ from Line 5.

[3] [Beck, Teboulle SIAM J. Imaging Sci (2009)],[Qi et al. CVPR (2016)]

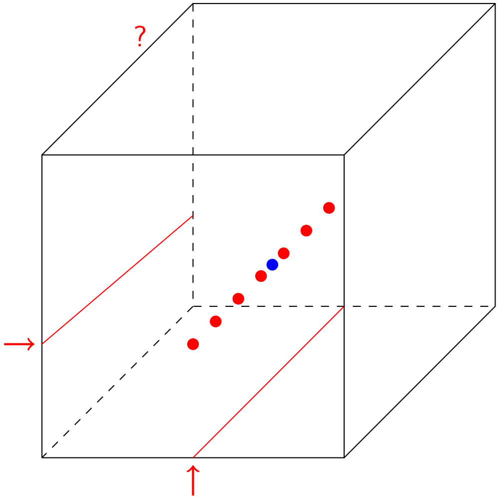
N-mode FISTA Shortcomings

1. May miss some support nodes, and compensate by erroneously assigning nearby nodes to the support.

2. Support values may be underestimated.

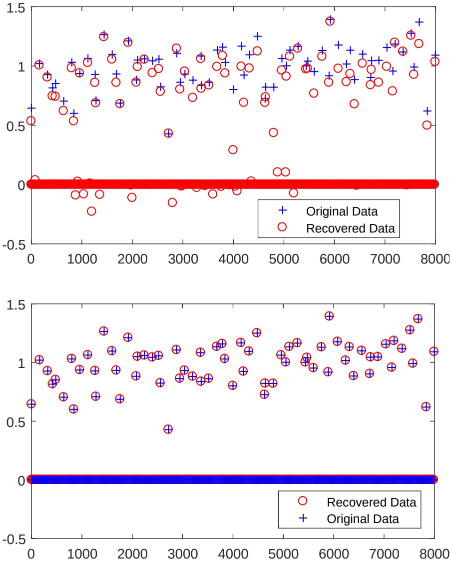


Postprocessing 1 - Fix support location errors



- Due to coherent dictionaries
- Identify when location errors occur
- Set all $X_t(\Omega) = 1$ in a neighborhood Ω of errors
- Restart iterations

Postprocessing 2 - Fix support value errors



Option 1:

$$\widetilde{\mathcal{X}}(\Omega) = \underset{u}{\operatorname{argmin}} \left\{ \|\mathcal{Y}_{\text{vec}} - P_{\Omega} u\|_F^2 \right\},$$

where $P = A_N \otimes A_{N-1} \otimes \cdots \otimes A_1$

Option 2:

Iterative method

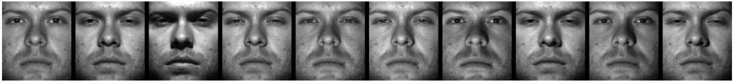
Experimental Results - Yale Faces

Recover $\mathcal{X}$ using N-FISTA with PP from $\mathcal{Y}_{\text{Original}} \approx \mathcal{X} \times_1 A_1 \times_2 A_2 \times_3 A_3$.

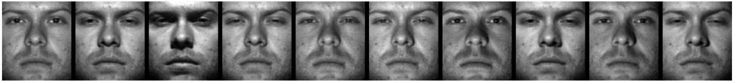
Yale Faces Results

$$\mathcal{Y}_{\text{Recovered}} := \mathcal{X} \times_1 A_1 \times_2 A_2 \times_3 A_3$$

Original



Recovered



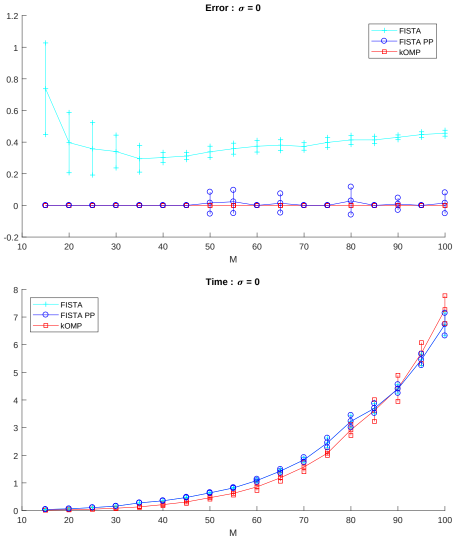
$$\frac{\|\mathcal{Y}_{\text{Original}} - \mathcal{Y}_{\text{Recovered}}\|_F}{\|\mathcal{Y}_{\text{Original}}\|_F} \approx 0.0352, \quad |\mathcal{X}|_0 \approx 4.95\%$$

Experimental Results - Synthetic Data

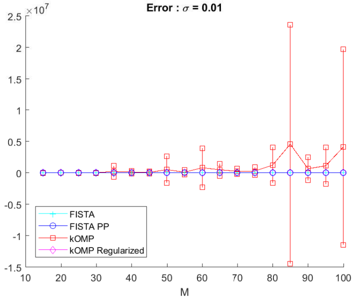
$$\mathcal{Y} = \mathcal{X} \times_1 A_1 \times_2 A_2 \times_3 A_3 + \mathcal{E}$$

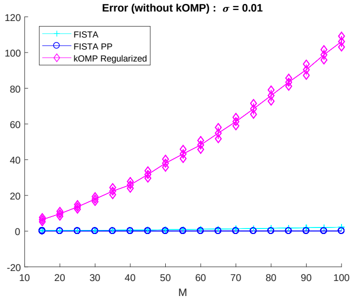
- $\mathcal{X} \in \mathbb{R}^{M \times M \times M}$, with $S = \lfloor 1.5M \rfloor$ nonzero entries
- $\mathcal{E} \sim N(0, \sigma)$
- A_1, A_2, A_3 randomly generated
- $M \in \{15, 20, 25, \ldots, 100\}$
- 20 simulations for each M
- Recovery methods:
  - Proposed N-mode FISTA with support correction
  - N-mode FISTA without support correction
  - Kronecker Orthogonal Matching Pursuit (Caiafa, Cichocki ICASSP 2012)
  - Kronecker Orthogonal Matching Pursuit with regularization

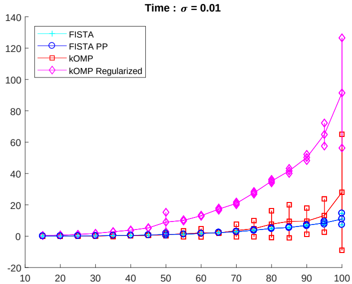
Noiseless Results



Noisy Results







Conclusion

- Support augmentation improved FISTA results
- Noiseless - similar to Kronecker OMP
- Robust to noise
- Future direction - learn factor matrices to avoid support errors

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