Robust Multi-User Analog Beamforming in mmWave MIMO Systems

Lisi Jiang
(with Hamid Jafarkhani)

UC Irvine
5G
• 1000 times the system capacity and 10 times the spectrum efficiency

mmWave
• Key technology
• High data rate and spectrum efficiency

MU-MIMO
• Higher system throughput

Background
MU-MIMO beamforming in mmWave

**Hybrid**
- Good performance
- Large feedback overhead

**Analog**
- Easy to implement and small feedback overhead
- No interference cancellation
- Advantageous over MU-MIMO

**Imperfect CSI**
- No existing robust analog beamforming design

**Our goal**
- Suppress interference and enhance beamforming gain simultaneously
- Provide robustness
Challenges and solutions

Tradeoff between interference and beamforming gain

- Establish Multi-objective problem
- Find the best weight assignment

Robust design

- Develop channel error model
- Introduce the stochastic approach
**System model**

**Signal model**

\[ y_i = h_i^H w_i s_i + \sum_{k=1, k \neq i}^{K} h_i^H w_k s_k + n_i \] (1)

**Leakage Interference**

\[ \text{Leakage} = \sum_{k=1, k \neq i} |h_k^H w_i|^2 = w_i^H \tilde{I}_i^H \tilde{I}_i w_i \] (2)
Clustered channel model

\[ h_i^H = \sqrt{\frac{N_t}{L}} \sum_{l=1}^{L} (a_l^i)^* \alpha_t(\theta_l^i)^H = \tilde{h}_i^H A_i^H \]

Array response vector for ULA

\[ \alpha_t(\theta_l^i) = \frac{1}{\sqrt{N_t}} [1, e^{j\frac{2\pi}{\lambda}d\sin(\theta_l^i)}, \ldots, e^{j(N_t-1)\frac{2\pi}{\lambda}d\sin(\theta_l^i)}]^T \]
Channel Error Model

\[ \alpha(\theta_i^i + \Delta \theta_i^i) = \frac{1}{\sqrt{N_t}} \left[ 1, e^{j\frac{2\pi}{\lambda} \sin(\theta_i^i + \Delta \theta_i^i)} , \ldots, e^{j(N_t-1)\frac{2\pi}{\lambda} \sin(\theta_i^i + \Delta \theta_i^i)} \right]^T \]  

\[ e^{j\kappa \sin(\theta_i^i + \Delta \theta_i^i)} = e^{j\kappa \sin(\theta_i^i)} + j\kappa \cos(\theta_i^i) \Delta \theta_i^i e^{j\kappa \sin(\theta_i^i)} \]  

\[ \tilde{\alpha}(\theta_i^i) = \alpha(\theta_i^i + \Delta \theta_i^i) \approx \alpha(\theta_i^i) + e_i^i \]

\[ A_i = A_i^p + E_i \]

\[ \tilde{I}_i = \tilde{I}_i^p + \tilde{E}_i \]
Problem Formulation

Interference suppression:

• Leakage Probability (restriction)

\[ P_{\text{leakage}} = \Pr \{ \mathbf{w}_i^H \mathbf{i}\tilde{i}_i^H \mathbf{i}_i \mathbf{w}_i \leq \gamma_i \} \] (9)

Average beamforming gain:

• Expectation

\[ B G_{\text{avg}} = E[ \mathbf{w}_i^H \mathbf{A}_i \mathbf{A}_i^H \mathbf{w}_i ] \] (10)
Problem formulation

\[ w_{i}^{opt} = \{ E[w_{i}^{H} A_i A_{i}^{H} w_i], Pr\{w_{i}^{H} \tilde{I}_{i}^{H} \tilde{I}_{i} w_i \leq \gamma_i\} \} \]

s.t. \( w_i \in \mathcal{W} \),

Challenge 1: The probability restriction

Challenge 2: How to deal with the MOP

Challenge 3: The non-convex constraints
Dealing with the probabilistic restriction

Using Markov’s inequality to transform the probabilistic restriction to a deterministic objective

\[
\Pr\{w_i^H \tilde{I}_i^H \tilde{I}_i w_i \leq \gamma_i\} = \Pr\{w_i^H (\tilde{I}_i^p + \tilde{E}_i)^H (\tilde{I}_i^p + \tilde{E}_i) w_i \leq \gamma_i\} \\
\geq 1 - \frac{E[w_i^H (\tilde{I}_i^p + \tilde{E}_i)^H (\tilde{I}_i^p + \tilde{E}_i) w_i]}{\gamma_i} \\
= 1 - \frac{\text{Tr}(((\tilde{I}_i^p)^H \tilde{I}_i^p + \tilde{C}_i) W)}{\gamma_i} \quad W = w_i w_i^H
\]

(12)
Problem reformulation

\[ W^{opt} = \left\{ \text{Tr}\left( (A_i^p(A_i^p)^H + C_i)W \right) \right\} \left( 1 - \frac{\text{Tr}\left( (\tilde{I}_i^p)^H\tilde{I}_i^p + \tilde{C}_iW \right)}{\gamma_i} \right) \]

s.t. \[ W_{ii} = \frac{1}{N_t}, \quad \forall i = 1, ..., N_t; \]
\[ W \succeq 0; \quad \text{rank}(W) = 1; \]

non-convex constraints

linear constraints

Multi-objective problem
Weighted-sum method

Semi-definite relaxing

Rank-one constraint
**SDP**

\[
SDP(W^{opt}) = \\{\lambda_1 \left( 1 - \frac{Tr((\tilde{I}^p_i)^H \tilde{I}^p_i + \tilde{C}_i)W)}{\gamma_i} \right) + \\
\lambda_2 Tr((A^p_i(A^p_i)^H + C_i)W)\} \\
\text{s.t. } W_{ii} = \frac{1}{N_t}, \quad \forall i = 1, \ldots, N_t; \\
W \succeq 0; \\
\text{rank}(W) = 1,
\]

**SDR**

\[
SDR(W^{opt}) = \\{\lambda_1 \left( 1 - \frac{Tr((\tilde{I}^p_i)^H \tilde{I}^p_i + \tilde{C}_i)W)}{\gamma_i} \right) + \\
\lambda_2 Tr((A^p_i(A^p_i)^H + C_i)W)\} \\
\text{s.t. } W_{ii} = \frac{1}{N_t}, \quad \forall i = 1, \ldots, N_t; \\
W \succeq 0.
\]

- SDR can be efficiently solved
- Approximation is needed
## Simulation

<table>
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<th>Methods</th>
<th>Imperfect channel model</th>
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<td>Fully-digital ZF</td>
<td>$\tilde{h}(A_i^p + E_i)$</td>
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<tr>
<td>Beam Selection</td>
<td>$\Delta \theta_i$ in beam alignment with mean 0 and variance $\sigma_i$</td>
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<tr>
<td>Our proposed method</td>
<td>$A_i^p + E_i$</td>
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• Strike a balance in terms of sum-rate
Performance comparison
Summery

Developed a channel error model for the scattering clustered channel model, which can serve as a general channel error model for mmWave channels.

Proposed a robust analog beamforming scheme for mmWave systems to alleviate the effects of the channel estimation and feedback quantization errors.

The proposed robust analog beamforming scheme brings about 109% improvement in sum-rate compared to the conventional beam selection method.
Thanks