A Neyman-Pearson Type Sensor Censoring Scheme for Compressive Distributed Sparse Signal Recovery

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Abstract

To strike a balance between energy efficiency and data quality control, this paper proposes a Neyman-Pearson type sensor censoring scheme for distributed sparse signal recovery via compressive-sensing based on wireless sensor networks. In the proposed approach, each sensor node employs a sparse sensing vector with known support for data compression, meanwhile enabling making local inference about the unknown support of the sparse signal vector of interest. We derive a closed-form formula of the optimal censoring rule; a low complexity implementation using bi-section search is also developed. Computer simulations are used to illustrate the performance of the proposed scheme.

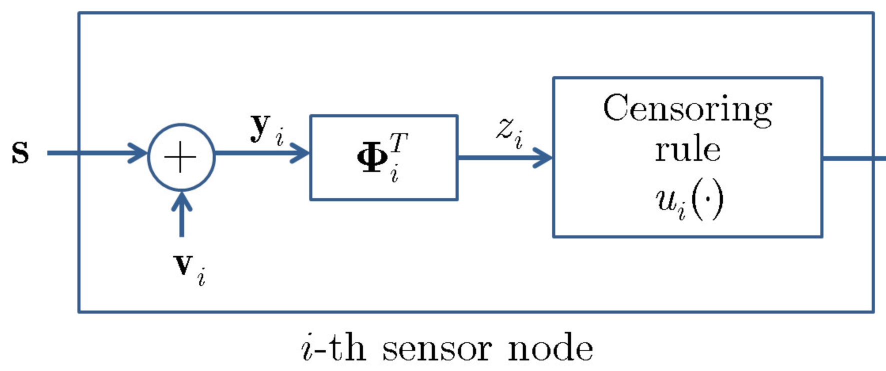
Motivation

- Among many physical-layer energy conservation schemes tailored for WSNs, sensor censoring, whereby only those sensors with good measurement quality are allowed to forward their data to the FC, was an effective means for reducing transmission energy.
- In CS-based WSNs, the measurement quality is even more demanding, because the global inference is made on the basis of relatively less data. This thus necessitates the development of sensor censoring schemes for CS- based WSNs, in an attempt to conserve transmission energy and, meanwhile, ensure availability of high-quality data to the FC.

Main Objectives

1. At each sensor (say, the ith node), a sparse sensing vector (with known support $\mathcal{A}_i$) is employed to compress the measurement vector, composed of a common desired sparse signal vector (with unknown support $\mathcal{T}$) contaminated by noise.
2. Thanks to the sparse nature of sensing vectors, each sensor can make its local inference based on the compressed measurement about the signal support (i.e., a local decision can be made to decide if $\mathcal{T} \cup \mathcal{A}_i$ is empty or not). i
3. Only those sensor nodes with reliable decisions are turned on and allowed to forward their censored decisions to the FC.
4. Our approach is essentially a ternary protocol: each sensor
   (a) directly transmits its compressed measurement if $T \cup \mathcal{A}_i \neq \emptyset$ is decided,
   (b) sends a one-bit hard decision when $T \cup \mathcal{A}_i = \emptyset$ is claimed,
   (c) keeps silent if the measurement is judged to be uninformative.
5. The design of our censoring scheme aims at minimizing the error probability of deciding $T \cup \mathcal{A}_i = \emptyset$ but $T \cup \mathcal{A}_i \neq \emptyset$ is true, subject to the constraints on (a) a tolerable false-alarm probability of deciding $T \cup \mathcal{A}_i \neq \emptyset$ while $T \cup \mathcal{A}_i = \emptyset$ is true, and (b) an allowable censoring rate.

System Model



$$u_i(z_i) = \begin{cases} z_i, & \text{if } \mathcal{T} \cap \mathcal{A}_i \neq \varnothing; \\ 0, & \text{if no decision is made;} \\ -1, & \text{if } \mathcal{T} \cap \mathcal{A}_i = \varnothing. \end{cases}$$

- M sensor nodes cooperate with a FC for estimating a K-sparse signal vector $\mathbf{s} \in \mathbb{R}^N$.
- s is supported on the unknown $\mathcal{T} \subset \{1, \ldots, N\}$ with $|\mathcal{T}| = K$ ($k \ll N$).
- $\mathbf{\Phi}_i \in R^N$ is K_c-sparse with support $A_i \subset \{1, \ldots, N\}$ ($K_c \geq K$).
- $\mathbf{v}_i \sim \mathcal{N}(\mathbf{0}_N, \sigma_v^2 \mathbf{I}_N)$.

Assumption 1: The signal support $\mathcal{T}$ is uniformly drawn from the collection $\mathbf{\Omega}_K := \{\mathcal{T}_1, \cdots, \mathcal{T}_{C_K^N}\}$ of all C_K^N possible sparsity pattern sets, where $\mathcal{T}_j \subset \{1, \ldots, N\}$ with $|\mathcal{T}_j| = K$ and $\Pr[\mathcal{T}_j] = 1/C_K^N$.

Assumption 2: The nonzero entries of s, say s_k, for $k \in \mathcal{T}$, are i.i.d. with $s_k \sim \mathcal{N}(0, \sigma_s^2)$, and are independent of the measurement noise $\mathbf{v}_i$.

Assumption 3: For each $1 \leq i \leq M$, the sensing vector $\mathbf{\Phi}_i$ is binary with K_c nonzero entries, i.e., $\mathbf{\Phi}_{ij} \in \{+1, -1\}$.

Assumption 4: The supports $\mathcal{A}_i$, $1 \leq i \leq M$, of the sensing vector Φ_i, $1 \leq i \leq M$, are known at the FC.

Problem Formulation

- The global signal recovery performance at the FC depends crucially on the reliability of the side information that can be inferred from the local sensor decisions.
- The local miss-detection probability, i.e., $P_M^{(i)} := \Pr[u_i(z_i) = -1 | T \cup \mathcal{A}_i \neq \emptyset]$, should be made as small as possible.
- The local false-alarm probability, i.e., $P_F^{(i)} := \Pr[u_i(z_i) = z_i | T \cup \mathcal{A}_i = \emptyset]$, should be constrained.
- The local censoring probability, i.e., $P_C^{(i)} := \Pr[u_i(z_i) = 0]$, cannot be set overly large.

We consider the following optimization problem.

$$(P1) \qquad \min_{u_i} P_M^{(i)} \text{ s.t. } P_C^{(i)} \leq \alpha_i, P_F^{(i)} \leq \beta_i$$

Censoring Rule

Let the LR for the measurement z_i be defined as $L(z_i) = \frac{p(z_i | T \cup \mathcal{A}_i \neq \emptyset)}{p(z_i | T \cup \mathcal{A}_i = \emptyset)}$.

Theorem 1: Let $L_0(\cdot) : \mathbb{R}^+ \cup \{0\} \rightarrow \mathbb{R}^+ \cup \{0\}$ be the restriction of $L(\cdot)$ on $\mathbb{R}^+ \cup \{0\}$, i.e., $= L|_{R^+ \cup \{0\}}$, and $L_0^{-1}(\cdot)$ be the corresponding inverse function. The optimal censoring rule u_i^*, which solves $(P1)$ can be expressed as

$$u_i^*(z_i) = \begin{cases} -1, & \text{if } |z_i| < \tau_1^{(i)}; \\ z_i, & \text{if } |z_i| > \tau_2^{(i)}; \\ 0, & \text{if } \tau_1^{(i)} < |z_i| < \tau_2^{(i)}, \end{cases}$$

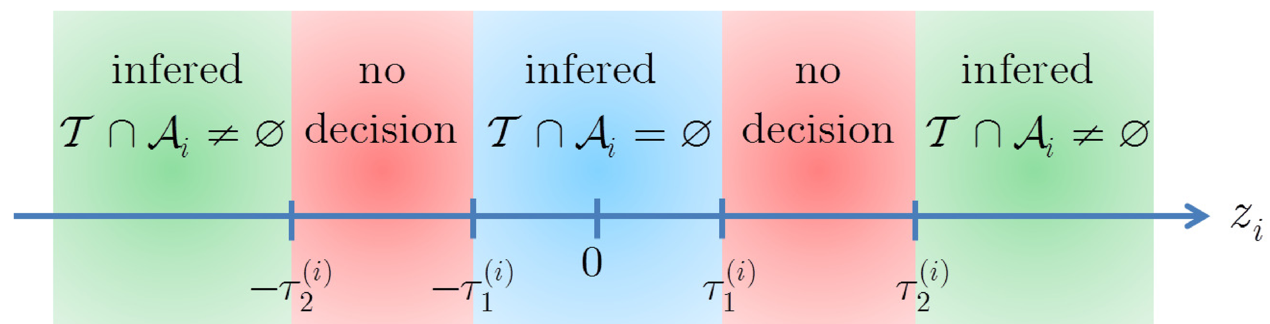
where $\tau_1^{(i)} := L_0^{-1}\left(\frac{\lambda_2^{(i)} \pi_0}{1 - \lambda_2^{(i)} \pi_1}\right)$, $\tau_2^{(i)} := L_0^{-1}\left(\frac{\lambda_1^{(i)} - \lambda_2^{(i)} \pi_0}{\lambda_2^{(i)} \pi_1}\right)$, $\pi_0 := C_K^{N-K_c}/C_K^N$, and $\pi_1 := \sum_{j=1}^K C_j^{K_c} C_{K-j}^{N-K_c}/C_K^N$.

Theorem 2: Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $g(x) = 2\pi_0 Q\left(\frac{x}{\sqrt{K_c}\sigma_v}\right) + 2\pi_1 \sum_{j=1}^K P_j Q\left(\frac{x}{\sqrt{j\sigma_s^2 + K_c \sigma_v^2}}\right)$, where $P_j = C_J^{K_c} C_{K-j}^{N-K_c} / \sum_{j=1}^K C_j^{K_c} C_{K-j}^{N-K_c}$ and $g^{-1}(\cdot)$ be the corresponding inverse function. Then we have

$$\tau_1^{(i)^*} = g^{-1}\left(\alpha_i + \pi_0 \beta_i + 2\pi_1 \sum_{j=1}^K P_j Q\left(\frac{\sqrt{K_c}\sigma_v Q^{-1}(\beta_i/2)}{\sqrt{j\sigma_s^2 + K_c \sigma_v^2}}\right)\right),$$

and

$$\tau_2^{(i)^*} = \sqrt{K_c}\sigma_v Q^{-1}(\beta_i/2).$$



Simulation Results

- The quality of signal recovery is evaluated by normalized mean square error (NMSE) defined as $NMSE := E[\|\mathbf{s} - \hat{\mathbf{s}}\| / \|\mathbf{s}\|]$.
- The energy efficiency is evaluated using the metric of the fraction of active nodes (FAN) defined as $FAN := \frac{\text{number of active nodes}}{\text{total number of sensors}}$.
- Performance comparison of the proposed scheme and the conventional CS method.
  - Conventional CS method (CS-ℓ_i), which activates all sensor nodes and utilized standard ℓ_1-minimization for signal reconstruction;
  - CS in conjunction with the proposed censoring scheme (CSC-ℓ_i), using standard ℓ_1-minimization for signal reconstruction