MMSE Adaptive Waveform Design for a MIMO Active Sensing System Tracking Multiple Moving Targets

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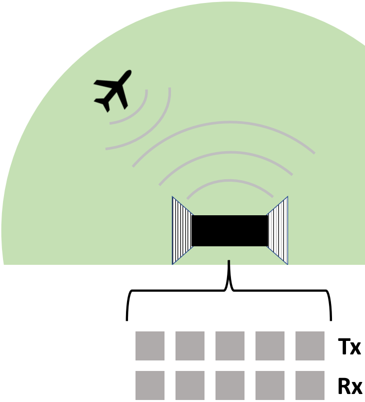
EPSRC
Engineering and Physical Sciences Research Council

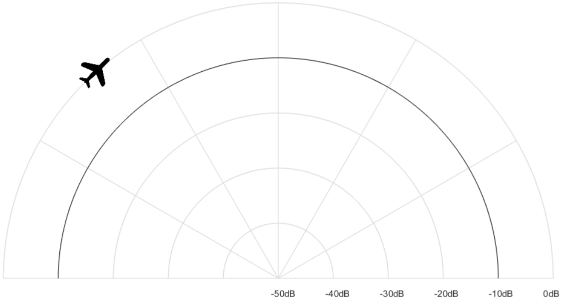
Presentation overview

- MIMO active sensing model
- Adaptive waveform design (AWD):
  - Problem statement
  - Our solution
- Generalisation for moving targets
- Conclusions

MIMO active sensing: radar example



AWD: the essential idea



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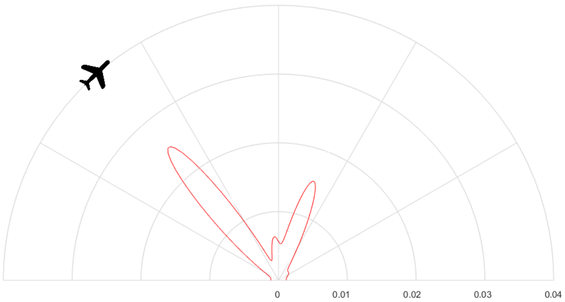

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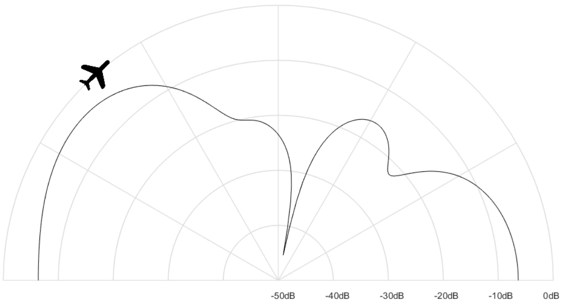

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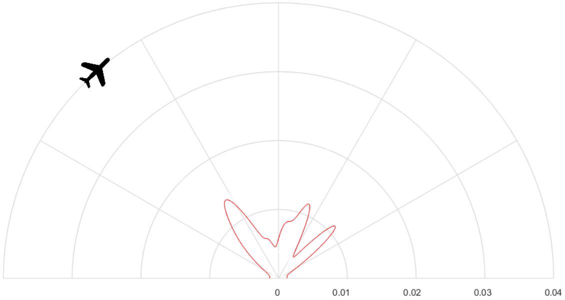

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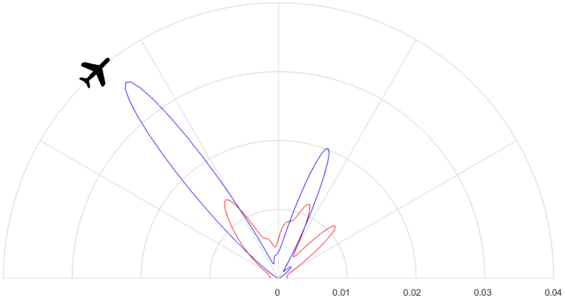
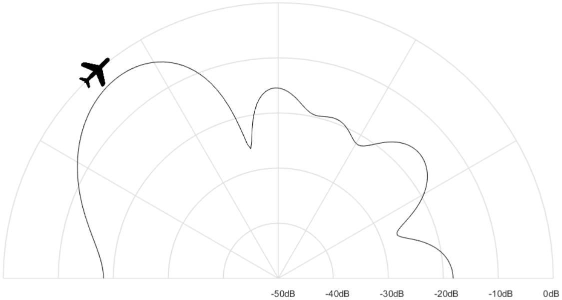


AWD: the essential idea



MIMO active sensing: system model

MIMO active sensing systems can be represented algebraically

$$\mathbf{X}_k = \mathbf{H}(\boldsymbol{\theta})\mathbf{S}_k + \mathbf{N}_k,$$

where $\boldsymbol{\theta} = [\phi; \Re(\alpha); \Im(\alpha)]$, and:

$$\mathbf{H} = \alpha \mathbf{a}_R(\phi)\mathbf{a}_T^T(\phi)$$

MMSE AWD: problem statement

$$\text{minimise: } \Sigma_k = \mathrm{tr}\left\{\mathbb{E}((\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta})^T | \mathbf{X}^{k-1})\right\} \quad \text{wrt } \mathbf{S}_k,$$

$$\text{subj. to: } \mathrm{tr}\left\{\frac{1}{L}\mathbf{S}_k\mathbf{S}_k^H\right\} \leq \mathrm{P},$$

where

$$\hat{\boldsymbol{\theta}}_k = \mathbb{E}(\boldsymbol{\theta}|\mathbf{X}^k, \mathbf{S}^k)$$

Expressing the MMSE AWD cost function was an open problem before our project.

MMSE AWD: our analytic solution

By definition

$$\Sigma_k = \iint (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta})^T (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}) p(\hat{\boldsymbol{\theta}}_k, \boldsymbol{\theta} | \mathbf{X}^{k-1}, \mathbf{S}^k) \, \mathrm{d}\hat{\boldsymbol{\theta}}_k \, \mathrm{d}\boldsymbol{\theta},$$

which we show can be rearranged

$$\Sigma_k = \iint (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta})^T (\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}) p(\boldsymbol{\theta} | \mathbf{X}^{k-1}, \mathbf{S}^{k-1}) p(\mathbf{X}_k | \boldsymbol{\theta}, \mathbf{S}_k) \, \mathrm{d}\mathbf{X}_k \, \mathrm{d}\boldsymbol{\theta}$$

MMSE AWD: our numerical solution

$\boldsymbol{\theta}$ estimated using a particle filter, which yields:

$$\Sigma_k \approx \Sigma'_k = \int \sum_{i=1}^{N_P} \left(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k^{(i)}\right)^T \left(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k^{(i)}\right) w_k^{(i)} p(\mathbf{X}_k | \boldsymbol{\theta}_k^{(i)}, \mathbf{S}_k) \, \mathrm{d}\mathbf{X}_k$$

where

$$\hat{\boldsymbol{\theta}}_k \approx \frac{\sum_{i=1}^{N_P} w_k^{(i)} p(\mathbf{X}_k | \boldsymbol{\theta}_k^{(i)}, \mathbf{S}_k) \boldsymbol{\theta}_k^{(i)}}{\sum_{i=1}^{N_P} w_k^{(i)} p(\mathbf{X}_k | \boldsymbol{\theta}_k^{(i)}, \mathbf{S}_k)}$$

... but we still have an integral and a sum

MMSE AWD: our numerical solution (cont.)

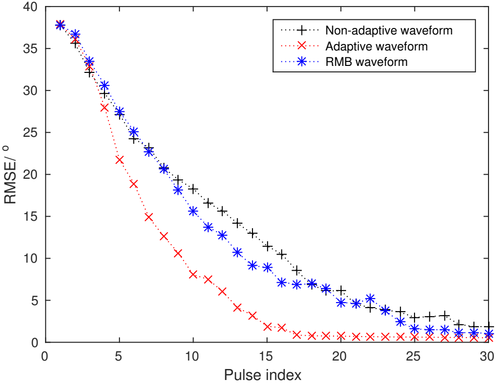
Solution, define a sample over the particles to do both sums in one go, for the mth sample:

$$\begin{array}{rcl} \boldsymbol{\theta}_k'^{(m)} & \sim & \sum_{i=1}^{N_P} w_k^{(i)} \delta(\boldsymbol{\theta}_k'^{(m)} - \boldsymbol{\theta}_k^{(i)}) \\ \mathbf{X}_k^{(m)} & \sim & p(\mathbf{X}_k^{(m)} | \boldsymbol{\theta}_k'^{(m)}, \mathbf{S}_k(0)) \end{array}$$

which leads to our final approximate cost function expression

$$\Sigma_k' \approx \Sigma_k'' = \sum_{m=1}^{N_S} \frac{p(\mathbf{X}_k^{(m)} | \boldsymbol{\theta}_k'^{(m)}, \mathbf{S}_k) / p(\mathbf{X}_k^{(m)} | \boldsymbol{\theta}_k'^{(m)}, \mathbf{S}_k(0))}{\sum_{m'=1}^{N_S} p(\mathbf{X}_k^{(m')} | \boldsymbol{\theta}_k'^{(m')}, \mathbf{S}_k) / p(\mathbf{X}_k^{(m')} | \boldsymbol{\theta}_k'^{(m')}, \mathbf{S}_k(0))} \times \left(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k'^{(m)}\right)^T \left(\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}_k'^{(m)}\right)$$

MMSE AWD: results



Generalisation to account for target motion

In general, the targets may be moving, thus we require:

- Statistical definition of actual target motion:

$$\boldsymbol{\theta}_k = \mathbf{f}(\boldsymbol{\theta}_{k-1}, \mathbf{v}_{k-1})$$

- System model of target motion:

$$\boldsymbol{\theta}_k = \tilde{\mathbf{f}}(\boldsymbol{\theta}_{k-1})$$

- ... leading to particle updates in the particle filter:

$$\boldsymbol{\theta}_k^{(i)} = \tilde{\mathbf{f}}(\boldsymbol{\theta}_{k-1}^{(i)})$$

Moving targets can improve particle filter performance

- Particle resampling
  - leveraging the standard particle filter technique such that the particles concentrate at regions of high probability density
- Sampling the particles to reduce complexity of expectation estimation
  - a contribution building on our previous work

Sampling the particles

Recall:

$$\Sigma''_k = \sum_{m=1}^{N_S} \frac{p(\mathbf{X}_k^{(m)}|\theta_k'^{(m)}, \mathbf{S}_k)/p(\mathbf{X}_k^{(m)}|\theta_k'^{(m)}, \mathbf{S}_k(0))}{\sum_{m'=1}^{N_S} p(\mathbf{X}_k^{(m')}|\theta_k'^{(m')}, \mathbf{S}_k)/p(\mathbf{X}_k^{(m')}|\theta_k'^{(m')}, \mathbf{S}_k(0))} \times \left(\hat{\theta}_k - \theta_k'^{(m)}\right)^T \left(\hat{\theta}_k - \theta_k'^{(m)}\right),$$

where

$$\hat{\theta}_k \approx \frac{\sum_{i=1}^{N_P} w_k^{(i)} p(\mathbf{X}_k|\theta_k^{(i)}, \mathbf{S}_k)\theta_k^{(i)}}{\sum_{i=1}^{N_P} w_k^{(i)} p(\mathbf{X}_k|\theta_k^{(i)}, \mathbf{S}_k)}.$$

We can use instead:

$$\hat{\theta}_k \approx \frac{\sum_{i=1}^{N_S} p(\mathbf{X}_k|\theta_k'^{(i)}, \mathbf{S}_k)\theta_k'^{(i)}}{\sum_{i=1}^{N_S} p(\mathbf{X}_k|\theta_k'^{(i)}, \mathbf{S}_k)}.$$

Computational complexity reduction

Before:

$$\mathcal{O}(N_S N_P (Q + L N_T N_R))$$

Now:

$$\mathcal{O}(N_S^2 (Q + L N_T N_R))$$

Simulation 1: matched model

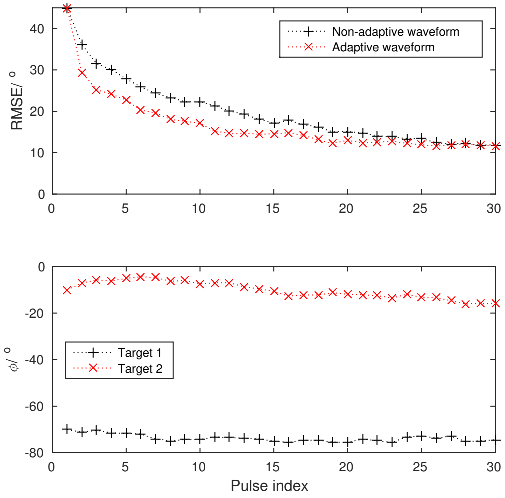
A random walk with no model mismatch:

- $\boldsymbol{\theta}_k = \mathbf{f}_1(\boldsymbol{\theta}_{k-1}, \mathbf{v}_{k-1}) = \boldsymbol{\theta}_{k-1} + \mathbf{v}_{k-1}$
- $\mathbf{v}_{k-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_2)$
- $\tilde{\mathbf{f}}_1 = \mathbf{f}_1$
- $\boldsymbol{\theta}_0 = [-70, -10]^T$.

General parameters:

- $N_T = N_R = 5,\ L = 1$
- $N_P = 1000,\ N_S = 250$
- $\text{ASNR} = 0$ dB
- Resampling threshold 90 %

Simulation 1: results



Simulation 2: mismatched model

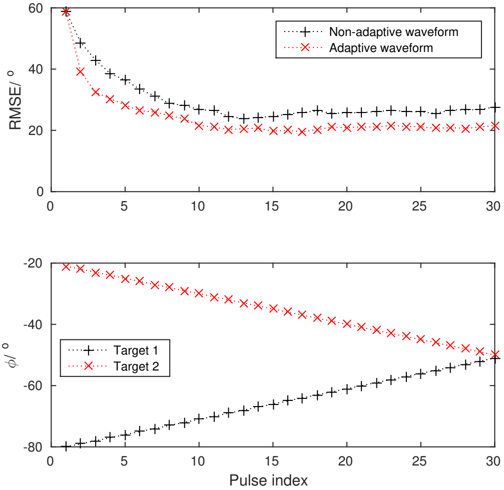
The targets move with constant angular velocity, but the MMSE AWD system treats the motion as a random walk:

- $\boldsymbol{\theta}_k = \mathbf{f}_2(\boldsymbol{\theta}_{k-1}) = \boldsymbol{\theta}_{k-1} + [1, -1]^T$
- $\tilde{\mathbf{f}}_2 = \tilde{\mathbf{f}}_1$
- $\boldsymbol{\theta}_0 = [-80, -21]^T$.

General parameters:

- $N_T = N_R = 5, \; L = 1$
- $N_P = 1000, \; N_S = 250$
- $\text{ASNR} = 0$ dB
- Resampling threshold 90 %

Simulation 2: results



Conclusions

- We have addressed MMSE AWD for multiple moving target tracking
- We have leveraged the fact that the targets *are* moving to use the standard particle filter technique of particle resampling
- We have shown that the existing samples can be used to provide a further computational saving
- We have presented numerical results that demonstrate that our AWD algorithm does indeed improve target parameter estimation both with and without a model mismatch

Contact and papers

Contact: S.Herbert@damtp.cam.ac.uk

Papers:

- S. Herbert, J. Hopgood, and B. Mulgrew, *MMSE Adaptive Waveform Design for a MIMO Active Sensing System Tracking Multiple Moving Targets*, ICASSP, 2018
- S. Herbert, J. Hopgood, and B. Mulgrew, *Optimality criteria for adaptive waveform design in MIMO radar systems*, SSPD 2017
- S. Herbert, J. Hopgood, and B. Mulgrew, *MMSE adaptive waveform design for active sensing with applications to MIMO radar*, IEEE transactions on signal processing 2017