LQG Control and Scheduling Co-design for Wireless Sensor and Actuator Networks

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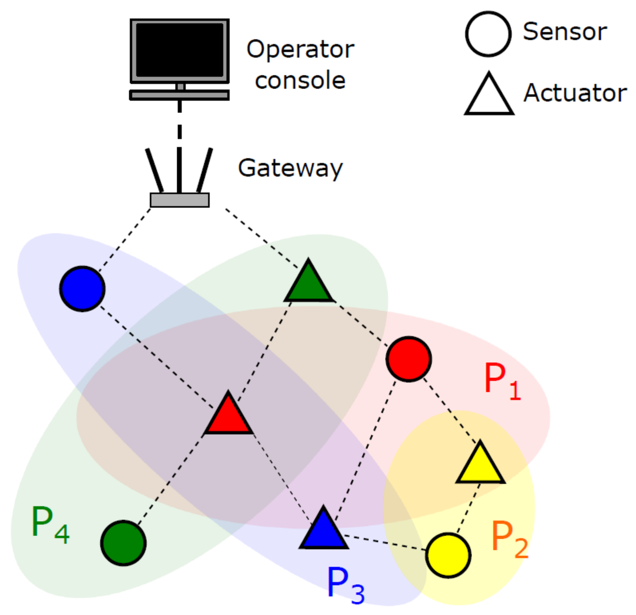
Abstract – We study a co-design problem of control, scheduling, and routing over a multi-hop sensor and actuator network (WSANs) subject to energy-saving consideration. We formulate an optimization problem, minimizing a linear combination of the averaged linear quadratic Gaussian (LQG) control performance and the averaged transmission energy consumption. Optimal solutions are derived and their performance is illustrated in a numerical example.

Motivation and Goals

Motivation

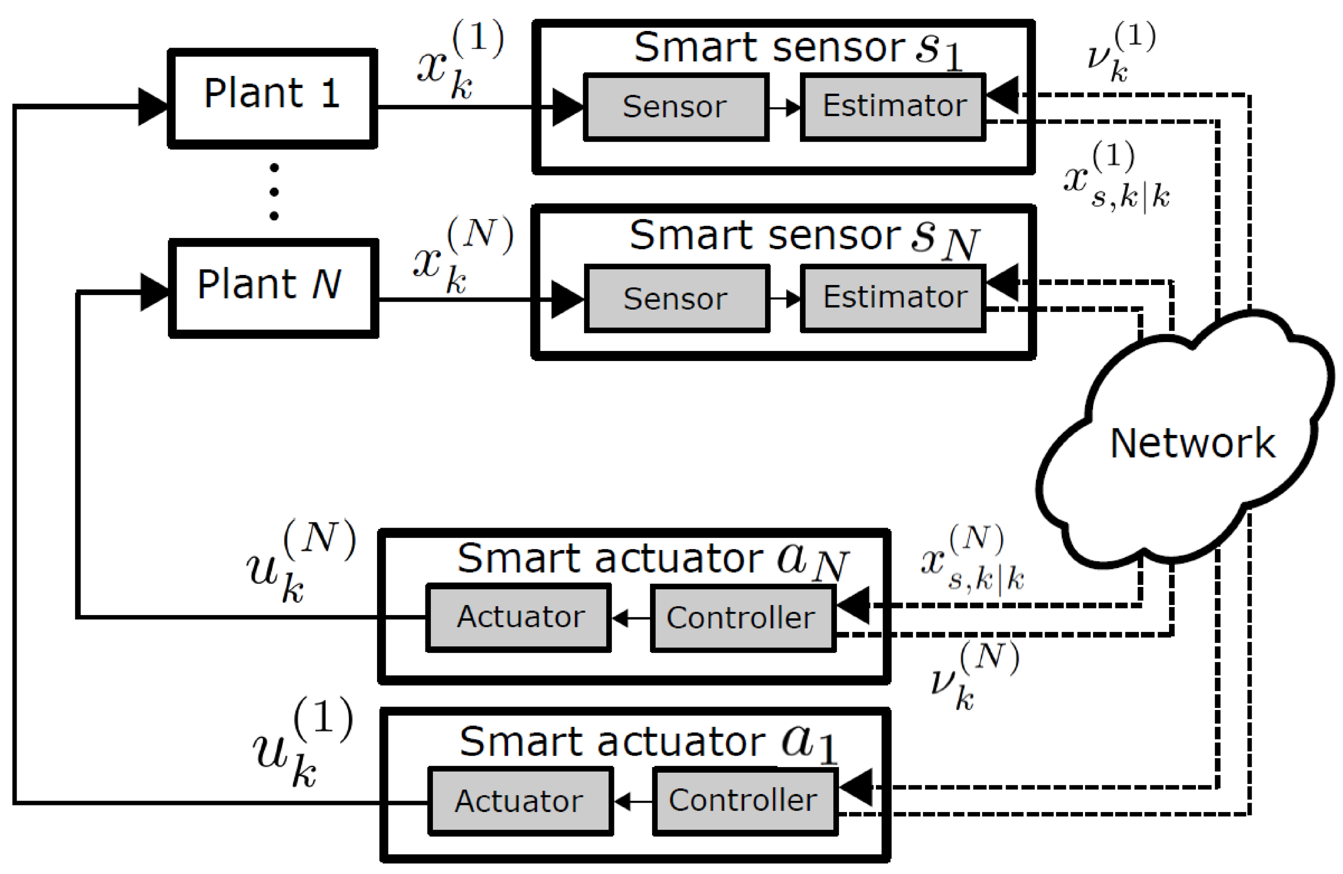
Smart sensors and actuators with co-located controllers are distributed over a field and communicating through a multi-hop network [1]. How can we find optimal control, scheduling, and routing to reduce energy consumption while maintaining a control performance?



Goals

Find a co-design framework of control, scheduling, and routing with LQG control gain by formulating an optimization problem.

System Model



Plant

$$x_{k+1}^{(i)} = A_i x_k^{(i)} + B_i u_k^{(i)} + w_k^{(i)}$$

Smart sensor

$$y_k^{(i)} = C_i x_k^{(i)} x_k^{(i)} + v_k^{(i)}$$

- $x_k^{(i)}$: process state
- $w_k^{(i)}$: process noise
- $y_k^{(i)}$: measurement
- $v_k^{(i)}$: measurement noise
- $x_{s,k|k}^{(i)}$: local Kalman estimate

Multi-hop Network

$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$

with $\mathcal{V} = \bigcup_{i=1}^{N} \{s_i, a_i\}, \ \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$

$\nu_k^{(i)} \in \{0,1\}$

: decision variable to send or not

$\theta_k^{(i)}((j,\ell)) : \mathcal{E} \to \{0,1\}$

: index variable if data i uses link (j,ℓ)

Energy consumption

$$E_{j,k} = \sum_{j:(\ell,j)\in\mathcal{E}} E_1 \sum_{i=1}^{N} c_i \theta_k^{(i)}((\ell,j)) + \sum_{j:(j,\ell)\in\mathcal{E}} E_2(j,\ell) \sum_{i=1}^{N} c_i \theta_k^{(i)}((j,\ell))$$

Optimization Problem

Optimization problem

$$\min_{\{\nu_k, u_k, \theta_k\}} \limsup_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} \left[\sum_{i=1}^{N} (x_k^{(i)\mathrm{T}} Q_i x_k^{(i)} + u_k^{(i)\mathrm{T}} R_i u_k^{(i)}) + \sum_{j\in\mathcal{V}} \beta_j E_{j,k} \right]$$

$$\text{s.t.} \quad \sum_{\ell:(j,\ell)\in\mathcal{E}} \theta_k^{(i)}((j,\ell)) - \sum_{\ell:(\ell,j)\in\mathcal{E}} \theta_k^{(i)}((\ell,j)) = 0, \qquad \text{if } \ell \neq s_i, a_i$$

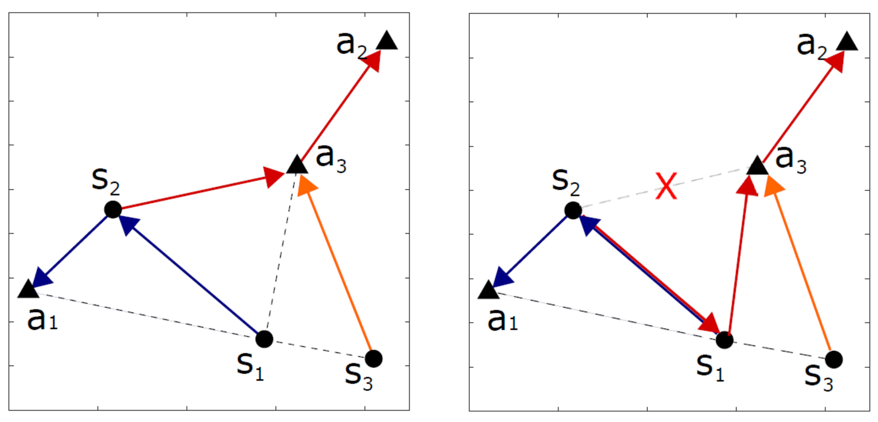
$$\sum_{\ell:(j,\ell)\in\mathcal{E}} \theta_k^{(i)}((j,\ell)) - \sum_{\ell:(\ell,j)\in\mathcal{E}} \theta_k^{(i)}((\ell,j)) = \nu_k^{(\ell)}, \qquad \text{if } \ell = s_i$$

$$\sum_{\ell:(j,\ell)\in\mathcal{E}} \theta_k^{(i)}((j,\ell)) - \sum_{\ell:(\ell,j)\in\mathcal{E}} \theta_k^{(i)}((\ell,j)) = -\nu_k^{(\ell)}, \qquad \text{if } \ell = a_i$$

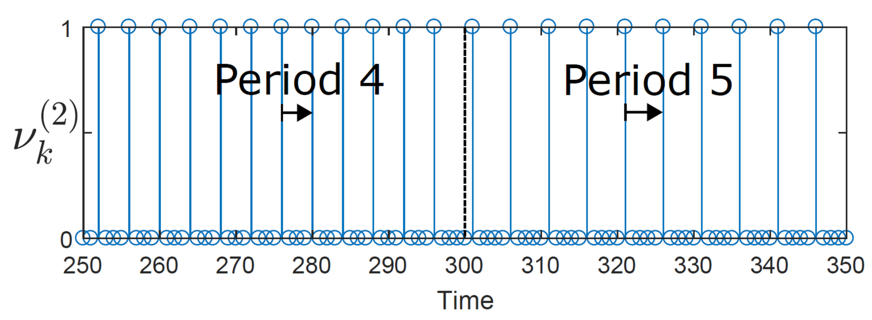
Theorem

1. **The Optimal control** is given by state feedback with standard LQG gain for each loop
2. **The Optimal schedule** is given by a covariance-based threshold policy [2] which results in a periodic schedule
3. **The Optimal routing** is given by solving minimum (energy) cost path problem for each loop

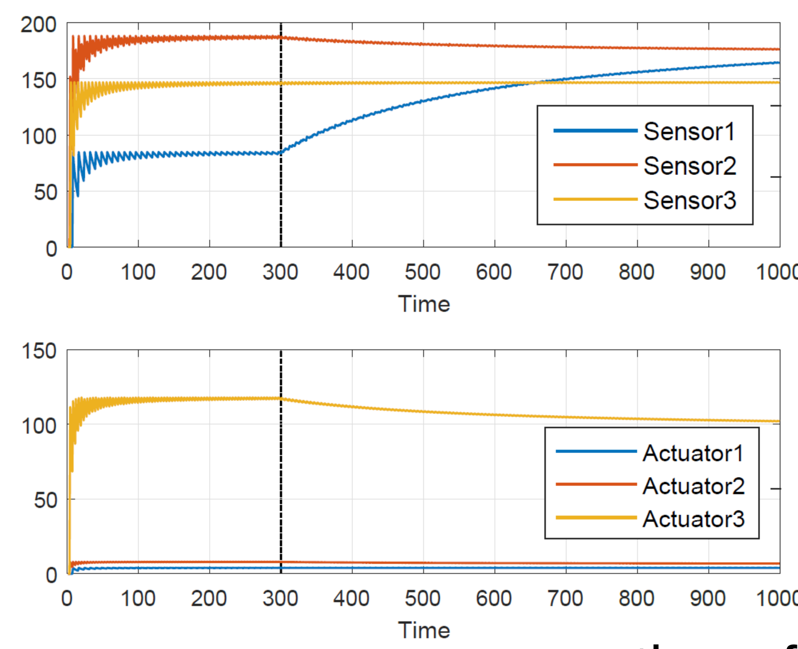
Numerical Example



WSAN of 3 control loops with the optimal routes (loop 1: blue, loop 2: red, loop 3: orange) before (left) and after (right) link disconnection between Sensor 2 and Actuator 3.



The optimal schedule for control loop 2. The optimal periodic schedule is obtained. The period is changed from period 4 to 5 due to link disconnection.



Average energy consumption of the sensors and the actuators.

Conclusion

We investigated the co-design framework of LQG control, sensor scheduling, and routing over a multi-hop WSAN. Future work includes WSANs with i) channel fading, ii) delay, and iii) constraints regarding specific protocol such as wirelessHART.

Reference

[1] A. J. Iskasson et al., “The impact of digitalization on the future of control and operations,” *Computers and Chemical Enginnering*, 2017

[2] A. S. Leong et al., “Event-based Transmission scheduling and LQG control over a packet dropping link,” in *Proc. IFAC World Congress*, pp. 8945-8950, 2017

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