Scheduling of Multistatic Sonobuoy Fields using Multi-Objective Optimization

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Outline

1. Multistatic Sonobuoy Fields
   - Two Tasks $\implies$ Search for and track underwater targets
   - Performance dependent on scheduling sonobuoys

2. Recap on Tracking in Sonobuoy Fields
   - Geometric Modelling and Measurements
   - Tracking algorithm used to track targets

3. Multi-Objective Scheduling Framework
   - Optimization Problem $\implies$ Two reward functions
   - Tracking Reward Function
   - Search Reward Function

4. Simulation Results

5. Conclusions
A network of transmitters and sensors distributed across a large search region
Two tasks of the system:

- Detect targets that are unknown to the system
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- Accurately track targets known to the system
Multistatic Sonobuoy Fields

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Scheduling Problem

- Choose sequence of transmitters and waveforms to satisfy tasks
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At one transmission time:

Choose a Transmitter: \( T = \{j_1, j_2, \ldots, j_{N_T}\} \)

where \( N_T \) is the number of transmitters in the field

Choose a Waveform: \( W = \{w_1, w_2, \ldots, w_{N_d}\} \)

where \( N_d \) is the number of possible waveforms
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Possible waveforms:

- Continuous Wave (CW) or Frequency Modulated (FM) waveform
- 1kHz or 2kHz frequency
- 2 second or 8 second duration
Scheduling Problem

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Action space:

Choose an action: \( a \in \mathcal{A}, \quad \mathcal{A} = \mathcal{T} \times \mathcal{W} \)
Conflicting Objectives

Track vs Search \rightarrow Which transmitter to choose...
Conflicting Objectives

Our Approach:

Combine both tasks in multi-objective framework and use multi-objective optimization to decide scheduling.
Sonobuoy Field Description:

- Transmitter positions
  \[ \mathbf{s}_j = \begin{bmatrix} x_j^s, y_j^s \end{bmatrix}^T \]

- Receiver positions
  \[ \mathbf{r}_i = \begin{bmatrix} x_i^r, y_i^r \end{bmatrix}^T \]

- Assume positions are known at all times*

‘x’ = Transmitters, ‘o’ = Receivers

*Each buoy contains RF communications and may contain GPS equipment
Target Description:

- Target Position at time $t_k$:
  \[ p = [x_k, y_k]^T \]

- Target Velocity at time $t_k$:
  \[ v = [\dot{x}_k, \dot{y}_k]^T \]

- Time-varying state
  \[ x_k = [p_k^T, v_k^T]^T \]

‘x’ = Transmitters, ‘o’ = Receivers
Target Motion:

- Noisy linear constant-velocity model

\[ \mathbf{x}_k = \left( \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \otimes \mathbf{I}_2 \right) \mathbf{x}_{k-1} + \mathbf{e}_k \]

- Process noise \( \mathbf{e}_k \) is Gaussian with variance

\[ \mathbf{Q} = \omega \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \otimes \mathbf{I}_2 \]

where \( T = t_k - t_{k-1} \) is the sampling in time

\( \otimes \) is the Kronecker product and \( \mathbf{I}_2 \) is \( 2 \times 2 \) identity matrix
Modelling, Measurements & Tracking Algorithm

Measurements:

- Signal amplitude $\beta$ and Kinematic measurement $z$
  \[ z = h_j^i(x_k) + w_j^i \]

- Measurements collected from a subset of receivers

- Buoys have two waveform modalities
  - Frequency Modulated (FM)
  - Continuous Wave (CW)

'x' = Transmitters, 'o' = Receivers
Using FM waveforms:

- Bistatic Range:
  \[ |p_k - r_i| + |p_k - s_j| \]

- Angle from Receiver:
  \[ \arctan \left( \frac{y_k - y_{r_i}}{x_k - x_{r_i}} \right) \]

- Good positional information

'x' = Transmitters, 'o' = Receivers
Using CW waveforms:

- **Bistatic Range:**
  \[ |\mathbf{p}_k - \mathbf{r}_i| + |\mathbf{p}_k - \mathbf{s}_j| \]

- **Angle from Receiver:**
  \[ \text{arctan} \left( \frac{y_k - y_{r_i}}{x_k - x_{r_i}} \right) \]

- **Bistatic Range-Rate:**
  \[ \mathbf{v}^T \left[ \frac{\mathbf{p}_k - \mathbf{r}_i}{|\mathbf{p}_k - \mathbf{r}_i|} + \frac{\mathbf{p}_k - \mathbf{s}_i}{|\mathbf{p}_k - \mathbf{s}_i|} \right] \]

- **Good velocity information**

‘x’ = Transmitters, ‘o’ = Receivers
Tracking Challenges:

- High levels of clutter
- Non-linear measurements
- Low probability of detection

'x' = Transmitters, 'o' = Receivers

Many possible algorithms: ML-PDA, MHT, PMHT, JIPDA, PHD/CPHD, ... etc
Modelling, Measurements & Tracking Algorithm

The tracker:

- Multi-Sensor Bernoulli filter\(^1\)
  (optimal multi-sensor Bayesian filter for a single target)

- Linear Multi-Target (LMT) Paradigm\(^2\)

- Gaussian mixture model implementation\(^3\)

- Process FM & CW measurements

`x` = Transmitters, `o` = Receivers

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Cite:

Multi-Objective Framework for choosing

Maximising rewards:

- \( R_{\text{Search}}(a) \Rightarrow \) Reward for searching to detect unknown targets
- \( R_{\text{Track}}(a) \Rightarrow \) Reward for continued tracking of known targets
Multi-Objective Framework for choosing

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Combine rewards via convex sum:

$$\max_a \left\{ \alpha R_{\text{Track}}(a) + (1 - \alpha) R_{\text{Search}}(a) \right\}$$

where $\alpha \in [0, 1]$
Multi-Objective Framework for choosing

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Performance depends on $\alpha \Rightarrow$ Controls trade-off
$\Rightarrow$ Different solutions depending on the value of $\alpha$
Multi-Objective Framework for choosing

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Pareto Optimality:

A point is Pareto optimal if there is no other point that can improve one objective without degrading the other.

Problem characterised $\Rightarrow$ Set of Pareto optimal points

$\Rightarrow$ Pareto Frontier
Given previous tracking:

† Measure the gain in tracking information from action \( a \)
Tracking Reward

Approximate information matrix:

Single track: 
\[
\text{trace} \left[ J_{\text{Predict}} + \sum_{i \in \mathcal{R}} P^i_d(a) J_{\text{Measure}}^i(a) \right]
\]

Trace of only the positional elements of information matrix

\( P^i_d(a) \) Expected probability of detecting track
**Tracking Reward**

**Predicted Information Matrix:**

\[
J_{\text{Predict}} = \left[ F_{k-1} P_{k-1} [F_{k-1}]^T \right]^{-1}
\]

Propagation of error covariance due to motion model

where \( F_{k-1} \) is the Jacobian of \( f(x_{k-1}) \) and \( P_{k-1} \) is the error covariance from tracker.
Tracking Reward

Measurement Information Matrix:

\[
J_{\text{Measure}} = \left[H_k^i(a)\right]^T \left[R_k^i(a)\right]^{-1} H_k^i(a)
\]

Gain in information from action

where \(H_k^i(a)\) is the Jacobian of \(h_a(x_{k-1})\) and \(R_k^i(a)\) is the measurement covariance
Multiple tracks:

$$R_{\text{Track}}(a) = \sum_{\tau=1}^{T} \omega_\tau \text{trace} \left[ J_{\text{Predict}}^\tau + \sum_{i \in \mathcal{R}} P_{d,i,\tau}^i(a) J_{\text{Measure}}^{i,\tau}(a) \right]$$

$$\omega_\tau \Rightarrow \text{Normalised weights (} \propto 1/\text{existence probability})$$
Search Reward

Reduction of the probability of undetected targets in sonar field
Reduction of the probability of undetected targets in sonar field

Modelling this probability$^1$:

- Define Threat Map $P_{T,k} \Rightarrow$ Discrete 2D grid of probabilities
- Probabilities evolve over time
  - Increases $\Rightarrow$ Drift & diffusion of undetected targets
  - Decreases $\Rightarrow$ Transmitters emits a ping

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Search Reward

Reduction of the probability of undetected targets in sonar field

Drift & diffusion process:

- Matrix $G \Rightarrow$ Probability of targets entering from adjacent cells
- Update to Threat Map $\Rightarrow$ Filter $P_{T,k}$ with $G$
- Pre-calculate $G$ using Monte-Carlo simulations

E.g. for a 60 s interval, grid size of 1 km, uniformly distributed target speed between 0 and 10 knots

$$G = \begin{bmatrix} 0.0036 & 0.0582 & 0.0036 \\ 0.0582 & 0.7526 & 0.0582 \\ 0.0036 & 0.0582 & 0.0036 \end{bmatrix}$$
Search Reward

Reduction of the probability of undetected targets in sonar field

Transmitting a ping:

Apply Bayesian update at each cell of $P_{T,k}$

$$P_{T,k}(x, a) = \frac{(1 - P_d(x, a))P_{T,k-1}(x)}{(1 - P_d(x, a))P_{T,k-1}(x) + (1 - P_{fa})(1 - P_{T,k-1}(x))}$$

- $P_d(x, a)$ is the probability a target is detected after action $a$
- $P_{fa}$ is the false alarm probability
- $x = (x, y)$ is the 2D grid point
Search Reward

Reduction of the probability of undetected targets in sonar field

Obtaining $P_d(x, a)$:

Generate probabilities using Monte-Carlo simulations and the realistic simulator (BRISE)

e.g.

- 160 × 160 km area
- 1km × 1km grid resolution
- 5 × 5 transmitter grid
- 6 × 6 receiver grid
- Buoy separation = 15km
- FM, 1 kHz waveform with 2 s duration.
Search Reward

Reduction of the probability of undetected targets in sonar field
and finally...

\[ R_{\text{search}}(a) = \sum_x P_{T,k-1}(x) - P_{T,k}(x, a) \]
Set-up:
- 4 × 4 transmitter grid
- 5 × 5 receiver grid
- Buoy separation = 15km
- 50 Minute Scenario
- 1 transmission/minute
- Blue target present for whole duration
- Green target appears after 10 minutes

‘x’ = Transmitters, ‘o’ = Receivers

Realistic measurements $\Rightarrow$ Bistatic Range Independent Signal Excess (BRISE) simulation environment
Analysis of Scheduler - Set Up

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Analyse the performance of the scheduler as $\alpha$ varies
α = 0.35
Analysis of Scheduler - Demo

\[ \alpha = 0.35 \]
Analysis of Scheduler - Results

Error bars = 95% confidence intervals for the estimated values
Red dashed line = Performance from random scheduling

Values averaged over 300 Monte-Carlo simulations and every transmission
Analysis of Scheduler - Results

Pareto-esque Frontier:

Values averaged over 300 Monte-Carlo simulations and every transmission
Analysis of Scheduler - Transmitter Choice

2D histogram showing the proportion of waveforms transmitted
Analysis of Scheduler - Transmitter Choice

2D histogram showing the proportion of waveforms transmitted

\( \alpha = 0 \)
Conclusions

- Introduced scheduling of multistatic sonobuoy fields
  - Search $\implies$ Detect targets that are unknown
  - Track $\implies$ Accurately track known targets

- Presented multi-objective framework for scheduling
  - Each task is treated as a separate objective
  - Objectives combined via weighted sum
  - Weight $\alpha$ controls priority placed on each objective

- Analysed proposed scheduling via realistic simulations
  - Demonstrated trade-off between search and track as $\alpha$ varies
  - Trade-off characterised in terms of points on the Pareto front
Thank you for listening