TS-MC: Two Stage Matrix Completion Algorithm for Wireless sensor networks

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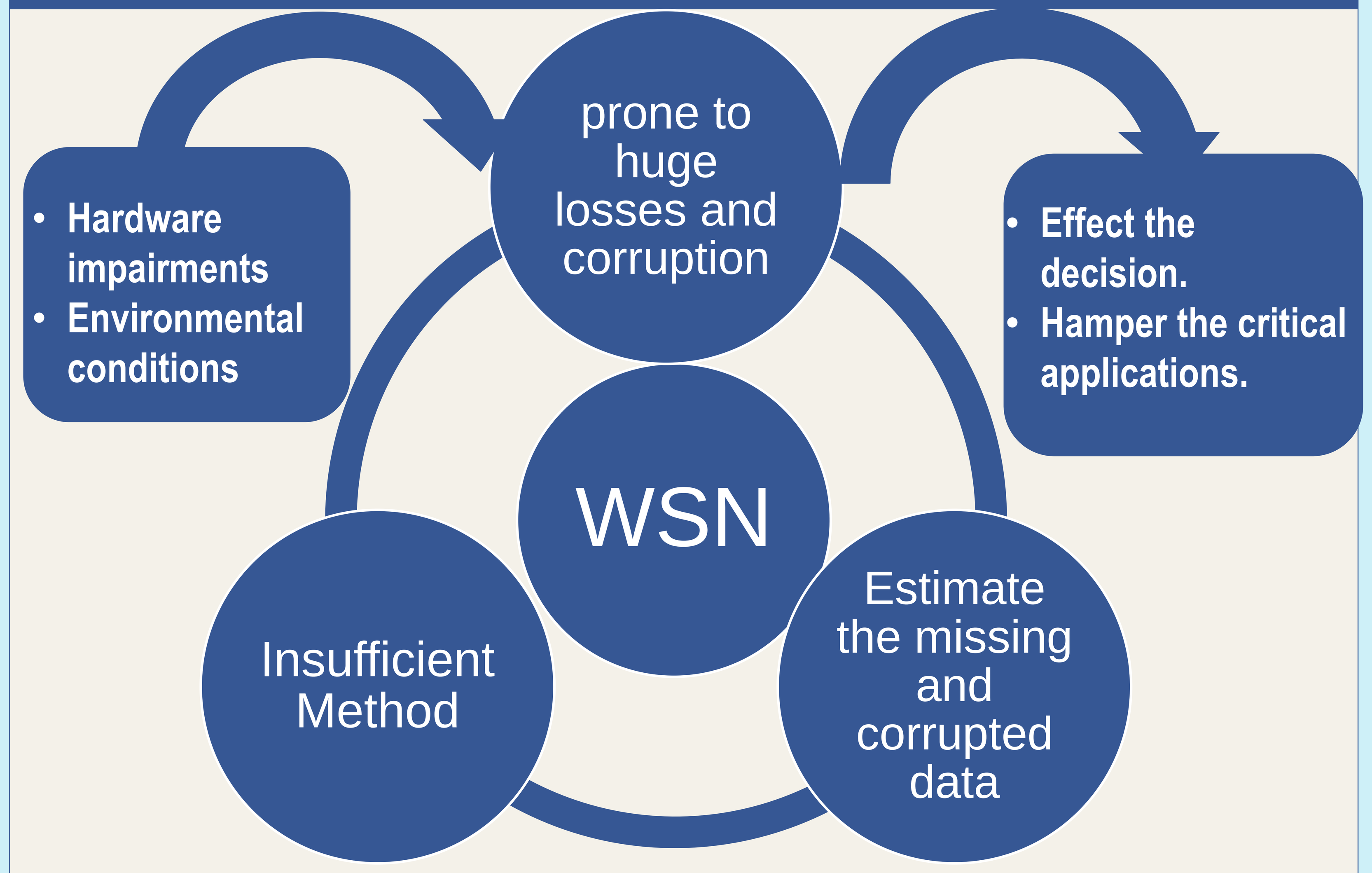
Motivation



Conventional Methods of Matrix completion

SVT: Singular value thresholding [1]

$$\min_{\mathbf{X}} ||\mathbf{X}||_* + \lambda_1 ||\mathbf{Y} - \mathbf{B} \bullet \mathbf{X}||_F^2$$

LMaFit: Low rank matrix fitting [2] = Matrix factorization with dynamically updated rank

$$\min_{\mathbf{X},\mathbf{U},\mathbf{V}} \lambda_2(||\mathbf{U}||_F^2 + ||\mathbf{V}^T||_F^2) + ||\mathbf{X} - \mathbf{U}\mathbf{V}^T||_F^2 + \lambda_3 ||\mathbf{Y} - \mathbf{B} \bullet \mathbf{X}||_F^2 \qquad \mathbf{U} \in \Re^{n \times r}, \ \mathbf{V} \in \Re^{t \times r}$$

RPCA: Robust principal component analysis [3]

$$\min_{\mathbf{X},\mathbf{L},\mathbf{S}} \lambda_4(||\mathbf{L}||_* + ||\mathbf{S}||_1) + ||\mathbf{X} - \mathbf{L} - \mathbf{S}||_F^2 + \lambda_5 ||\mathbf{Y} - \mathbf{B} \bullet \mathbf{X}||_F^2$$

Data loss reconstruction in WSN [4]

$$\min_{\mathbf{X},\mathbf{U},\mathbf{V}} \lambda_6(||\mathbf{U}||_F^2 + ||\mathbf{V}^T||_F^2 + ||\mathbf{H}\mathbf{U}\mathbf{V}^T||_F^2 + ||\mathbf{U}\mathbf{V}^T\mathbf{T}||_F^2) + ||\mathbf{X} - \mathbf{U}\mathbf{V}^T||_F^2 + \lambda_7 ||\mathbf{Y} - \mathbf{B} \bullet \mathbf{X}||_F^2$$

$\mathbf{X} \in \Re^{n \times t}$ is unknown WSN matrix, with *n* sensor nodes and *t* timestamps

$\mathbf{Y} \in \Re^{n \times t}$ is known incomplete and noisy WSN matrix

$$B(i,j) = \begin{cases} 0 & \text{if } Y(i,j) = 0 \\ 1 & \text{Otherwise} \end{cases}$$

Proposed: TS-MC Algorithm

First Stage

- ***Method-1***

$$\min_{\hat{\mathbf{X}}} ||\mathbf{Y} - \mathbf{B} \bullet \mathbf{X}||_F^2 + \lambda_{p1} ||\mathbf{D}_1 \hat{\mathbf{X}} \mathbf{D}_2||_1$$

$\mathbf{D}_1 \in \mathbb{C}^{n \times n}$, $\mathbf{D}_2 \in \mathbb{C}^{t \times t}$ are the DCT matrices.

- ***Method-2***

Matrix completion problem formulated as Compressive sensing problem.

$$\mathbf{y} = \mathbf{\Phi}\mathbf{X}(:) = \mathbf{\Phi}\mathbf{x} = \mathbf{\Phi}(\mathbf{D}_2 \otimes \mathbf{D}_1^T)\mathbf{s} = \mathbf{\Phi}\mathbf{\Psi}^{-1}\mathbf{s}$$

$$\mu(\mathbf{\Phi}, \mathbf{\Psi}^{-1}) = \sqrt{nt} \max_{\forall i,j} \frac{|\langle \mathbf{\Phi}_i, \mathbf{\Psi}_j^{-1} \rangle|}{||\mathbf{\Phi}_i||_2 \, ||\mathbf{\Psi}_j^{-1}||_2} = \sqrt{2},$$

$$\mu(\mathbf{\Phi}, \mathbf{\Psi}^{-1}) \in [1, \sqrt{nt}]$$

$$\hat{\mathbf{s}} = \min_{\mathbf{s}} ||\mathbf{y} - \mathbf{A}\mathbf{s}||_2^2 + \lambda_c ||\mathbf{s}||_1, \quad \hat{\mathbf{x}} = \mathbf{\Psi}^{-1}\hat{\mathbf{s}}$$

- WSN data varies slowly/ smoothly in both spatial and temporal domain.
- DCT acts as a KL type basis for large class of smooth signals, hence may act as the best sparsifying basis for WSN signal in both spatial and temporal domain.

Second Stage

Compute rank of the data from First stage

$$\min_{\mathbf{U},\mathbf{V}} ||\hat{\mathbf{X}} - \mathbf{U}\mathbf{V}||_F^2 + \mu ||\mathbf{U}||_F^2 + \mu ||\mathbf{V}||_F^2$$

$$\tilde{\mathbf{X}} = \mathbf{U}\mathbf{V}$$

Proposed TS-MC-1: Method-1 + Second stage

Proposed TS-MC-2: Method-2 + Second stage

Simulation Results

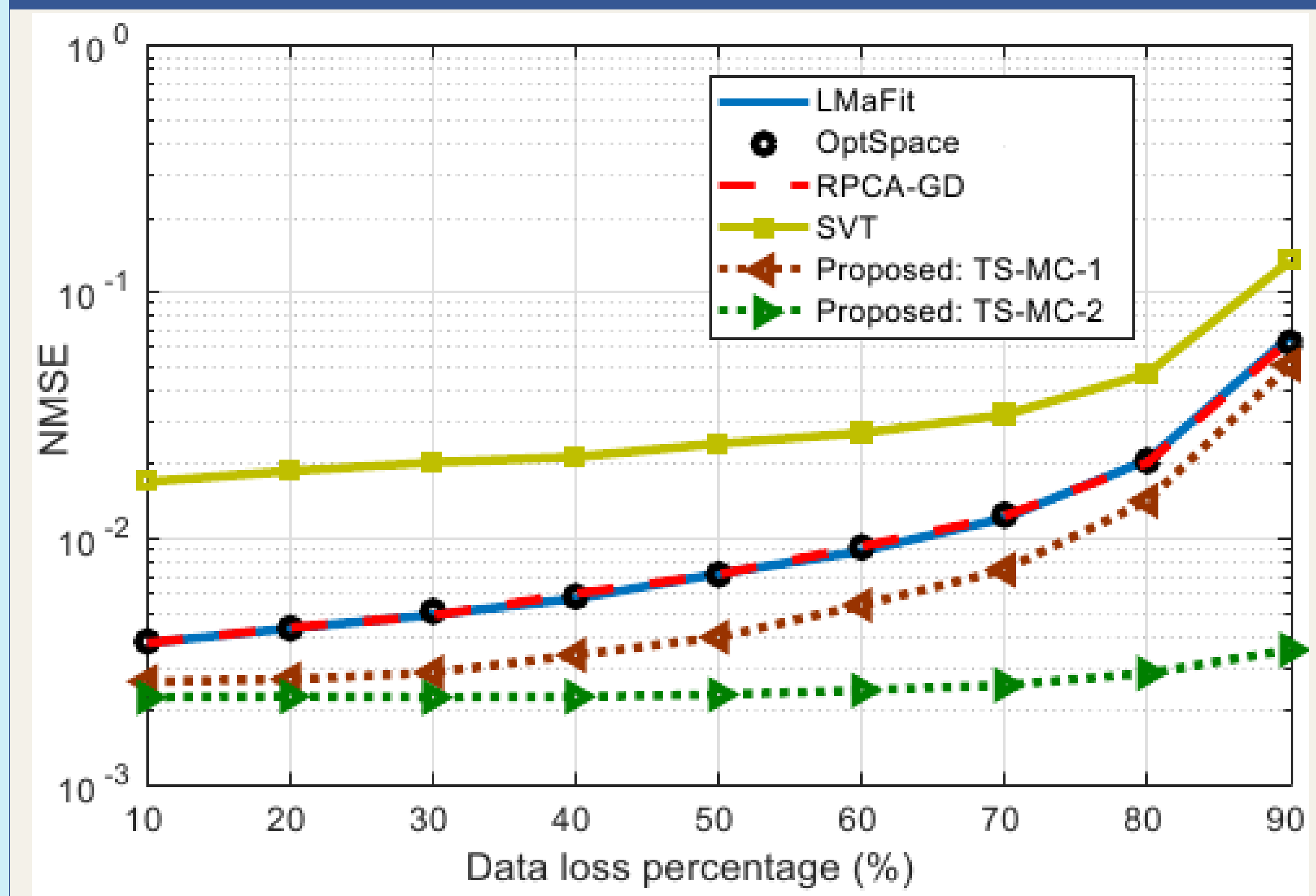


Fig. 1 NMSE against data loss percentage at SNR = 10 dB for humidity dataset taken from Intel lab [6].

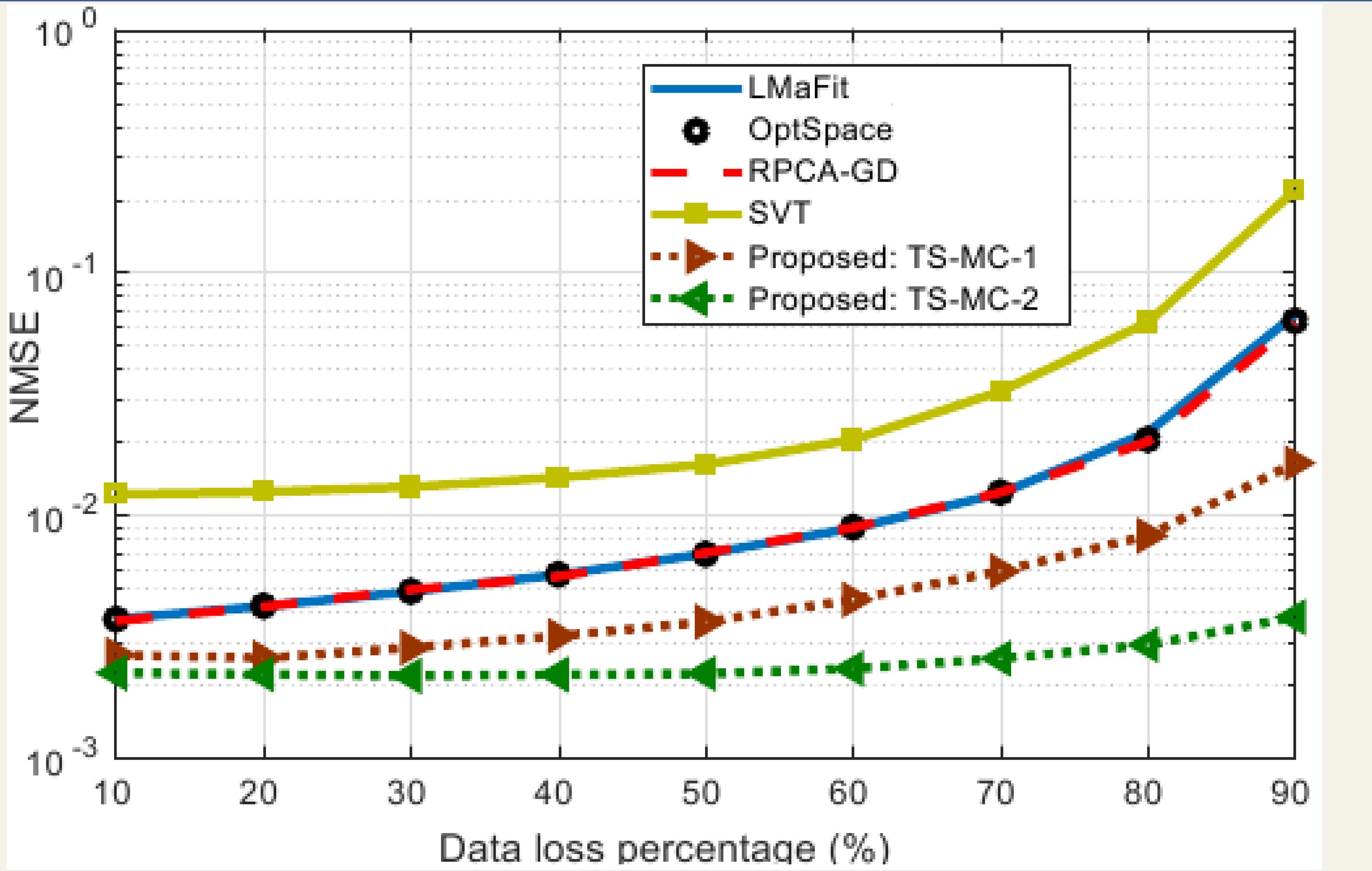


Fig. 2 NMSE against data loss percentage at SNR = 10 dB for temperature dataset taken from Intel lab [6].

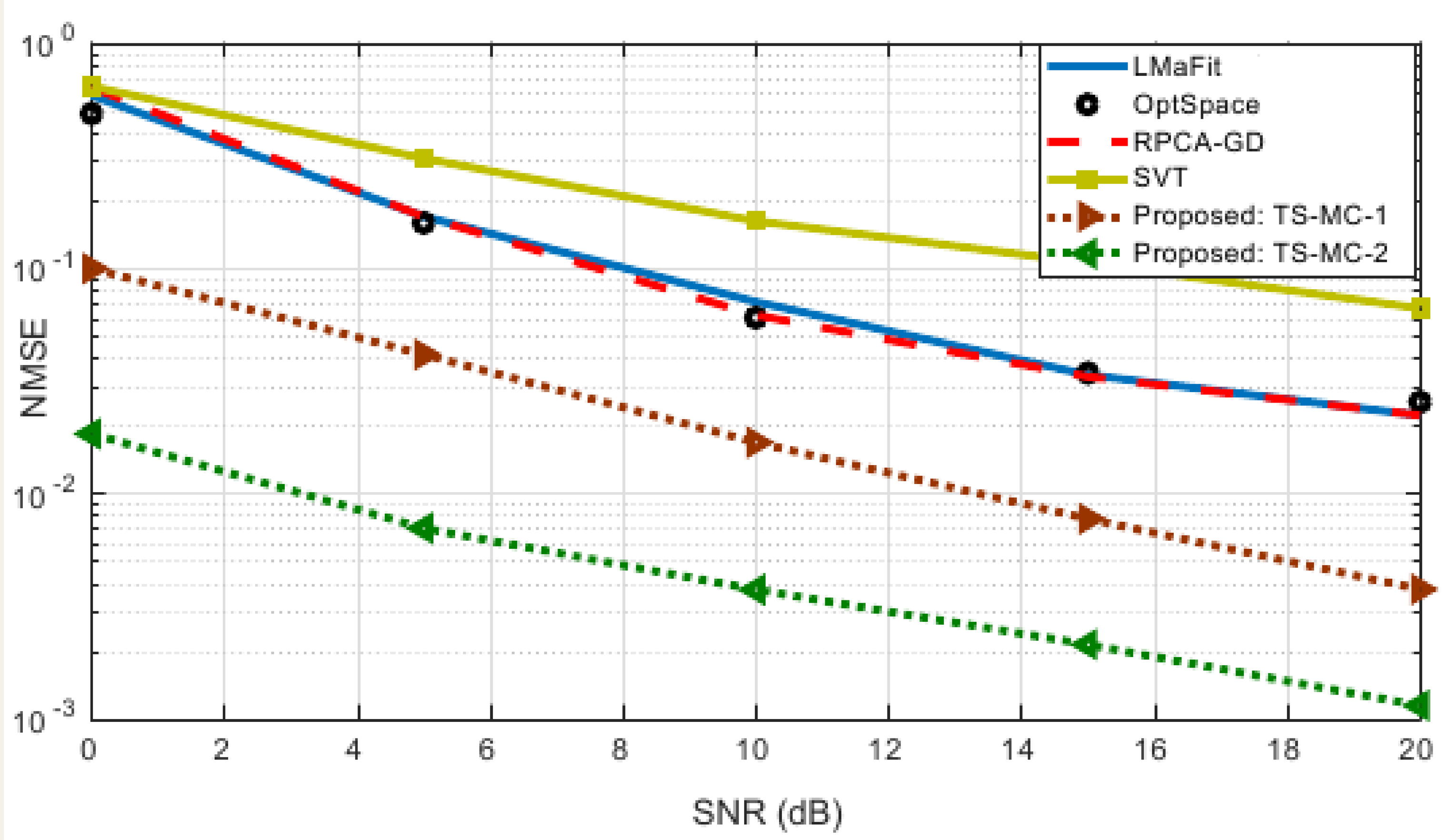


Fig. 3 NMSE against SNR (dB) at 90% data loss for temperature dataset taken from Intel lab [6].

Future Work

- The proposed TS-MC algorithm will be verified for various other data sets like biology data and recommender system.
- This algorithm will also be used for channel estimation in millimeter wave Massive MIMO system.

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