Robust 3D Localization of Partially Connected Underwater Optical Wireless Sensor Networks via Low Rank Matrix Completion

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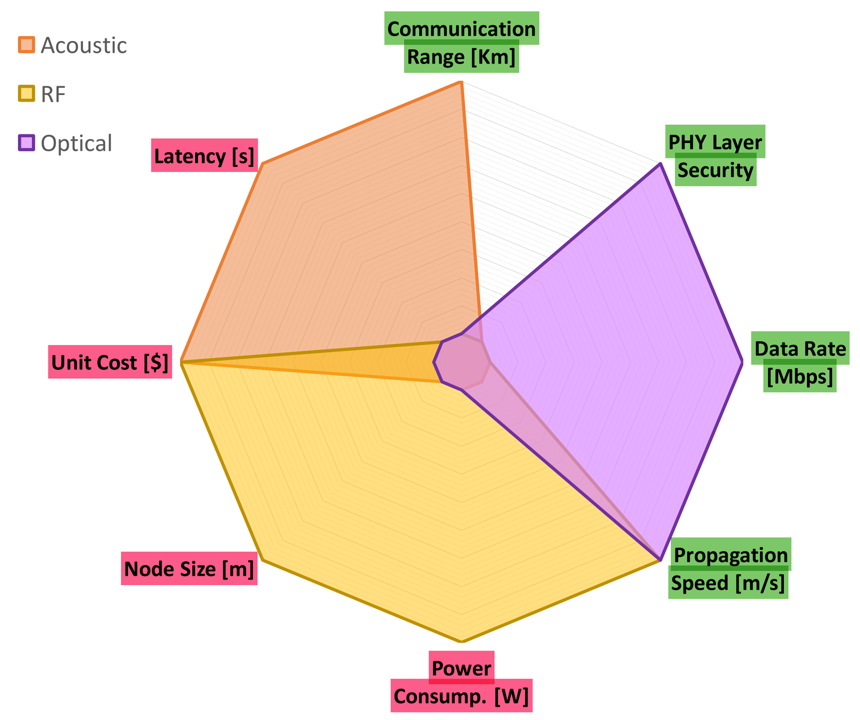
Problem Statement

1. The high quality of service demand for underwater exploration necessitates high data rate, low latency, and long-range networking solutions.

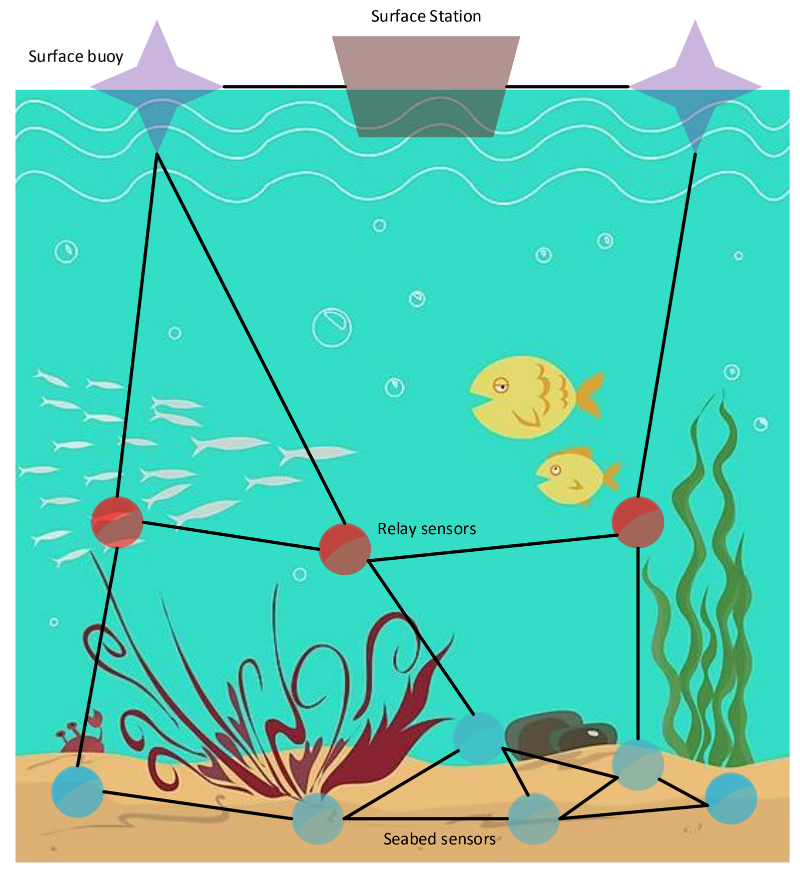


2. In particular, we are interested in the localization UWOSN nodes as the collected data is useful only if it refers to a particular location.

Main Contributions

1. Two-dimensional localization methods for UOWSNs have been studied in the literature [1, 2]. To the best of our knowledge, this paper is first to consider 3D localization for UOWSNs.
2. Due to the directivity and limited transmission range of UOWC, there are many missing inter-node distances, which are required to be accurately estimated. Hence, we develop a low-rank matrix approximation method which can precisely estimate the missing inter-node distances.
3. Some of the inter-node distance can have a large error and introduces outliers to which the conventional 3D network localization methods are quite susceptible. Consequently, a closed-form convergent iterative solution is proposed which can accommodate the negative impacts of outliers.

Network Model



The Euclidean distance between any two arbitrary nodes i and j is given by

$$d_{ij}(\mathbf{P}) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}, \quad (1)$$

and the corresponding matrix of measured pairwise squared distances is denoted as with outliers is given by

$$\hat{\boldsymbol{D}} = \begin{bmatrix} 0 & \hat{d}_{12} & ? & \cdots & \hat{d}_{1N} \\ \hat{d}_{21} & 0 & o_{23} & \cdots & \hat{d}_{2N} \\ \hat{d}_{31} & o_{32} & 0 & \cdots & o_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{d}_{N1} & ? & o_{N3} & \cdots & 0 \end{bmatrix}, \quad (2)$$

where ? are the missing pairwise distances and o_{ij} represents the outliers.

Proposed Localization Method

Least square loss function which measures the raw stress between $\hat{d}_{ij}$ and d_{ij}, i.e.,

$$\sigma(\boldsymbol{P}) = \sum_{i<j}^{N} w_{ij}\left(\hat{d}_{ij} - d_{ij}(\boldsymbol{P})\right)^2, \quad (3)$$

where w_{ij} represents the user-defined nonnegative weights to give importance to the measured distances. Even if the problem defined in (3) is a non-convex optimization problem without a unique solution, it can be solved by iterative majorization approach which minimizes

$$\sigma(\boldsymbol{P}) = \| -\frac{1}{2}\boldsymbol{C}[\hat{\boldsymbol{D}}^2 - \boldsymbol{D}^2]\boldsymbol{C} \|_F^2, \quad (4)$$

However, l_1 norm is not smooth due to the fact that it has a singularity at its origin. To mitigate this problem, the use of Huber's loss function can be of great benefit which interpolates between l_1 and l_2 norms minimizations, i.e.,

$$\sigma_h(\boldsymbol{P}) = \sum_{i<j}^{N} w_{ij}\gamma\left(\hat{d}_{ij} - d_{ij}(\boldsymbol{P})\right)^2, \quad (5)$$

where γ is the Huber's loss function,

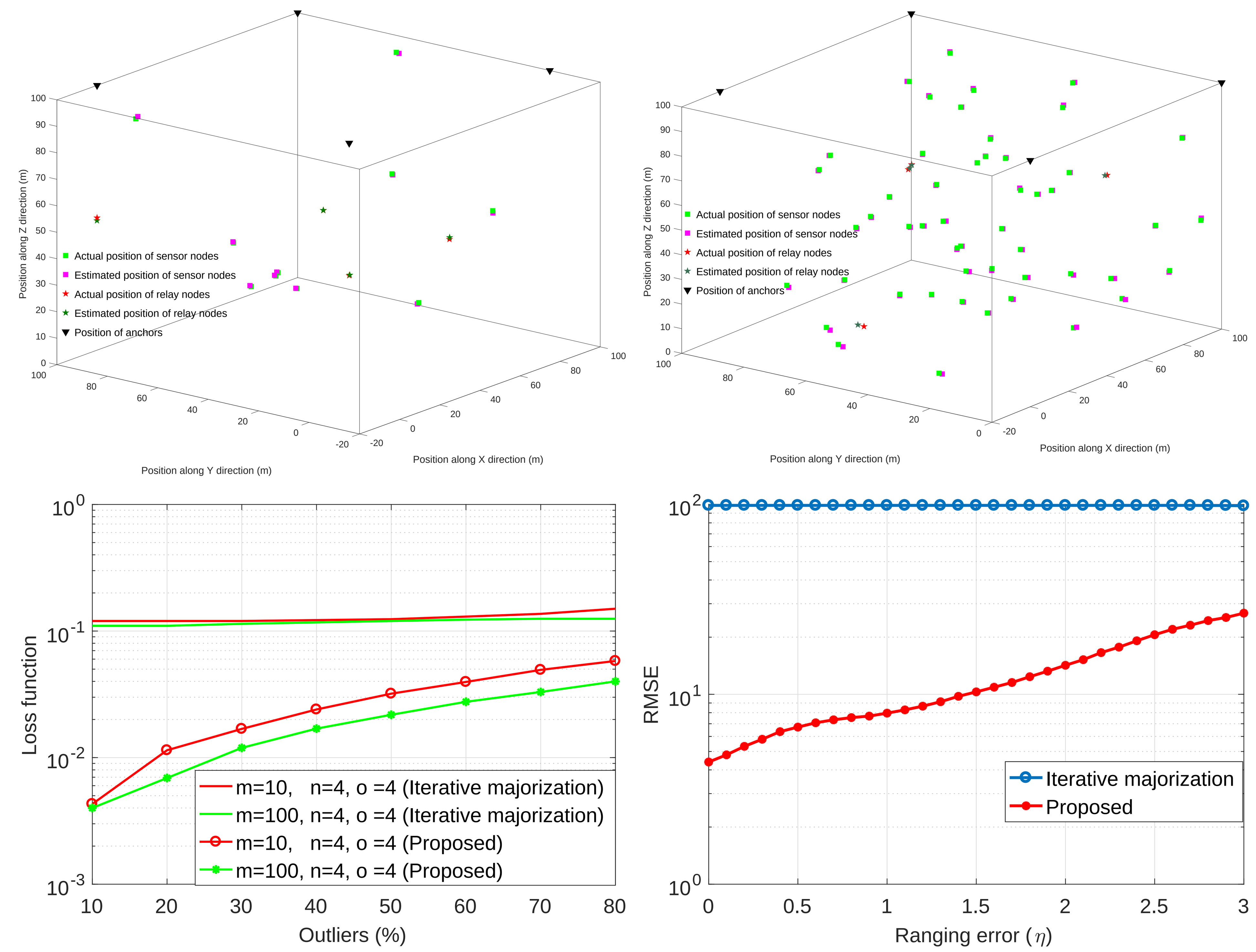
$$\gamma = \begin{cases} \frac{a^2}{2} & \text{if } |a| \le \rho \\ \rho|a| - \frac{\rho^2}{2} & \text{if } |a| > \rho \end{cases}, \quad (6)$$

a is the residual error, and ρ is the threshold which can be chosen adaptively or arbitrarily from matrix $\hat{\boldsymbol{D}}^2$. Therefore, inserting the l_1 norm minimization problem into the l_2 norm minimization problem in (3) yields

$$(\hat{\boldsymbol{P}}, \hat{\boldsymbol{O}}) = \min_{\boldsymbol{P},\boldsymbol{O}} \left\{ \sum_{i<j}^{N} w_{ij}\left(\hat{d}_{ij} - d_{ij}(\boldsymbol{P}) - o_{ij}\right)^2 + \lambda_1 \sum_{i<j}^{N} |o_{ij}| \right\}. \quad (7)$$

where the first term in (7) corresponds to the level of fitness between $\hat{d}_{ij}$ and d_{ij} after removing the outlier o_{ij} and the second term corresponds to the penalty linked to the sparsity of matrix $\boldsymbol{O}$ where λ_1 represents the regularization parameter. Iterative majorization and minimzation can be used to solve (7).

Results



Area: $100 \times 100\ m^2$, # Nodes: $10 - 100$, # Anchors: 4, Ranging error: $0.1 - 3\ m$, and Range: $50 - 80$ m.

Conclusions

In this paper, we have proposed a robust 3D localization method for UOWSNs with limited connectivity. As the transmission distance of underwater optical sensors is limited, it leads to a partially connected network and many of inter-node distances are missing. Hence, we have employed a low-rank matrix approximation method which can accurately estimate the missing inter-node distances. Additionally, some of the estimated inter-node distances may have a large error and naturally introduces outliers. The traditional 3D network localization methods are susceptible to these outliers. Consequently, a closed-form convergent iterative solution is proposed which can accommodate these outliers.

References

[1] F. Akhoundi et. al., "Underwater positioning system based on cellular underwater wireless optical CDMA networks," in *Wireless and Optical Communication Conference (WOCC)*, Apr. 2017, pp. 1–3.

[2] N. Saeed et. al., "Underwater optical sensor networks localization with limited connectivity," in *IEEE Int. Conf. on Acoustics, Speech and Signal Processing (ICASSP)*, Apr. 2018, pp. 1–5.