On the Behavior of Delay Network Reverberator Modes

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Objectives

- Find the modal decomposition of Delay Network Reverberators using a state space formulation.
- Introduce a parameterized orthogonal mixing matrix which can be continuously varied from identity to Hadamard.
- Investigate the effect of smoothly varying mixing matrix on frequency and damping of modes of Delay Network Reverberators, including Feedback Delay Network (FDN) reverberators.
- Quantify the perceptual effect of increasing mixing by calculating the Normalized Echo Density (NED) of Feedback Delay Network (FDN) impulse responses over time.
- Derive an empirical relationship between mixing matrix, average delay line length and mixing time of the FDN reverberator.

Delay Network Reverberators

- Used to synthesize reverberation.
- Impulse response has a set of sparse early reflections with increasing density over time.
- Feedback delay networks have a unitary feedback matrix (mixing matrix), \( M \), to mix the outputs of the delay lines [1].
- FDN reverberators have delay line filters, \( \varphi_i(z) \), to yield a single desired frequency dependent \( T_{60} \) response [2].
- Delay Network Reverberator design parameters are mixing matrix, shelf filters and number and length of delay lines.

Modal Decomposition

- State space equations:

\[
\begin{align*}
\dot{x}(n) &= Ax(n-1) + bu(n) \\
\dot{y}(n) &= c^T x(n) + du(n)
\end{align*}
\]

- Modes of the DNR are the eigenvalues of the state transition matrix \( A \).
- \( A \) is a sparse block matrix whose order is the sum of delay line lengths. The sub-matrix sizes are given by the delay line lengths plus the associated filter order.

Mixing Matrix Effect

\[
R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}
\]

\[
M(\theta) = I + \sum \theta \otimes \theta 
\]

- Defined as the time when NED reaches the threshold of 0.9.
- Schlecht [5] has a mechanism to determine the mixing time given a mean delay line length for a random orthogonal mixing matrix.
- Here we have a closed form expression for mixing time vs. angle, given a mean delay line length.

Decay Filter Design

- In FDN reverberators, decay filters are designed such that all delay lines independently produce the same \( T_{60} \) frequency response.
- Physical rooms produce multiple, concurrent \( T_{60} \) responses.
- One set of filters can model air absorption, another set of filters can model absorption by materials in the room.
- Mixing matrix can emulate occupancy or clutter of a room, or coupled rooms.

References


Conclusion

- Explicitly derived the modal decomposition of delay network reverberators.
- Described how mixing affects modal behavior, NED profile and mixing time.
- Derived closed-form expression for mixing time vs. angle.