Adaptive prediction of financial time-series for decision-making using a tensorial aggregation approach

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Outline

1. Introduction;
2. Proposed approach;
3. Numerical experiments;
4. Conclusion.

Introduction - MCDA problem

multiple criteria decision analysis (MCDA) methods are applied to rank, select or classify a set of alternatives according to multiple criteria:

$$\mathbf{A} = \{a_1, a_2, \ldots, a_m\}, \mathbf{C} = \{c_1, c_2, \ldots, c_n\}.$$

Central element: *decision matrix* $\mathbf{P} \in \mathbb{R}^{m \times n}$:

$$\mathbf{P} = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \ldots & c_n \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & \ldots & p_{1n} \\ p_{21} & p_{22} & \ldots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \ldots & p_{mn} \end{bmatrix} \end{array}.$$

Each element p_{ij} represents the evaluation of criterion j in alternative i.

Introduction - MCDA problem

Matrix *aggregation*:

$$\mathbf{P} \in \mathbb{R}^{m \times n} \Rightarrow \mathbf{g} \in \mathbb{R}^{m}$$

g: overall value for ranking the alternatives.

Classical examples: weighted mean, PROMETHEE II, Choquet integral, etc.

Introduction - Example of an economic-financial decision problem

To choose a country to invest, based on two criteria: *gross domestic products* (GDP) and *purchasing power parity* (PPP).

- Alternatives: Country 1, Country 2
- Criteria: GDP, PPP

Decision matrix $\mathbf{P}$:

$$\begin{array}{c} \\ Country\ 1 \\ Country\ 2 \end{array} \begin{array}{c} \begin{array}{cc} GDP & PPP \end{array} \\ \begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix} \end{array}$$

Usual approach: to consider only *current information*, represent by t_T, to build matrix $\mathbf{P}$.

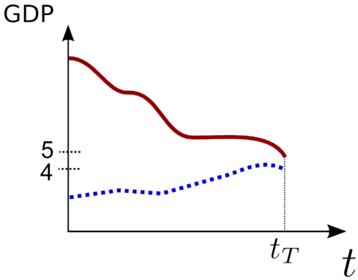
Introduction - Example

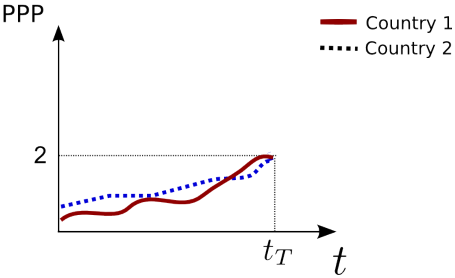
Country 1 is chosen due to its superiority of GDP.

$$\begin{array}{lc} & \begin{array}{cc} GDP & PPP \end{array} \\ \begin{array}{l} Country\ 1 \\ Country\ 2 \end{array} & \begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix} \end{array}$$

Introduction - Example

What if we observe the time series of the criteria instead of the current data?

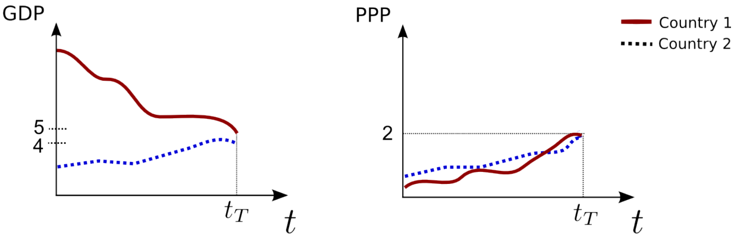




	GDP	*PPP*
Country 1	5	2
Country 2	4	2

Introduction - Example

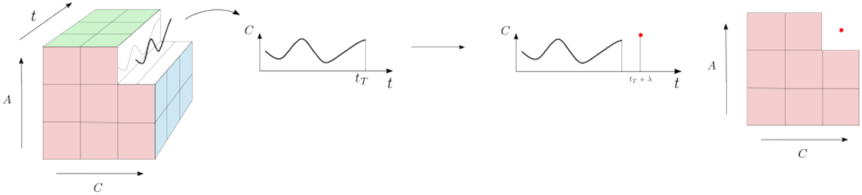
- When there is a long-term (or medium-term) impact by the decision made, the analysis of time-series becomes relevant.



Introduction - Tensorial approach

The base of our proposal is to introduce a *Tensorial* approach in MCDA problems and to apply an adaptive prediction method:

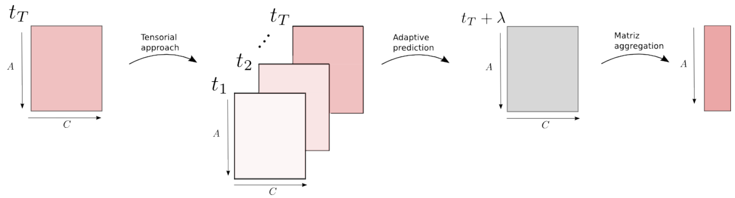
Decision tensor, $\mathcal{P} \in \mathbb{R}^{m \times n \times T}$:



Tensorial approach is not widely used in MCDA problems

Introduction - Proposed approach

$$\mathbf{P} \in \mathbb{R}^{m \times n} \Rightarrow \mathcal{P} \in \mathbb{R}^{m \times n \times T} \Rightarrow \hat{\mathbf{P}} \in \mathbb{R}^{m \times n} \Rightarrow \hat{\mathbf{g}} \in \mathbb{R}^{m}$$



- The adaptive prediction algorithm: recursive least square (RLS) and normalized least-mean-square (NLMS).
- The matrix aggregation method: PROMETHEE II.

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Proposed approach - Algorithm steps

Consider $\mathbf{x}(t) = \mathbf{p}(m, n, :) = [p_{mn(t-1)}, p_{mn(t-2)}, \cdots, p_{mn(t-M)}]$. M is the number of parameters $\mathbf{w}$. The desired signal is $x(t+\lambda)$, and λ is the step of prediction.

Algorithm 1 Tensor aggregation with NLMS algorithm

Input: $\mathcal{P} \in \mathbb{R}^{m \times n \times T}$ and weights $\gamma \in \mathbb{R}^n$
Initialization: $\mathbf{x}(0) = \mathbf{w}(0) = [0, 0, \cdots, 0]^T$; $0 < \mu \leq 1$, and $\theta \geq 0$ a small constant

for $i = 1$ *to* m **do**
  for $j = 1$ *to* n **do**
    $\mathbf{x}(t) \leftarrow \mathbf{p}(m, n, :)$
    for $t = 1$ *to* T **do**
      $y(t) = \mathbf{x}^T(t)\mathbf{w}(t)$
      $e(t) \leftarrow x(t+\lambda) - y(t)$
      $\mathbf{w}(t+1) \leftarrow \mathbf{w}(t) + \frac{\mu}{\theta + \mathbf{x}^T(t)\mathbf{x}(t)} e(t)\mathbf{x}(t)$
    end for
    $p_{ij} \leftarrow \mathbf{x}^T(t)\mathbf{w}(t+1)$
  end for
  $\mathbf{p}_i \leftarrow p_{ij}$
end for
$\mathbf{P} \leftarrow \mathbf{p}_i$
$\hat{\mathbf{g}} \leftarrow$ **Solve** PROMETHEE II method using input $\mathbf{P}$ and γ
Output: $\hat{\mathbf{g}}$

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Kendall tau rank correlation coefficient

- For the numerical experiments it is necessary to measure the difference between two rankings.
- We use the Kendall tau rank correlation coefficient $\tau_{\mathbf{g}\times\hat{\mathbf{g}}}$, which evaluates the distance between two rankings: $\mathbf{g}$ and $\hat{\mathbf{g}}$.

$0 \leq \tau_{\mathbf{g}\times\hat{\mathbf{g}}} \leq 1$;
$\tau_{\mathbf{g}\times\hat{\mathbf{g}}} = 0 \iff$ the rankings are equivalent.
$\tau_{\mathbf{g}\times\hat{\mathbf{g}}} = 1 \iff$ the rankings are different.

Numerical experiments

Let us consider the following rankings:

- $\hat{\mathbf{g}}$: ranking obtained by our proposal;
- $\mathbf{g}^*$: ranking obtained by future data (assumed to be known);
- $\mathbf{g}^c$: ranking obtained by current data.

Numerical experiments

We shall consider two Kendall tau distances:

- $\tau_{\mathbf{g}^c \times \mathbf{g}^*}$: to compare the approaches using **current data** and **future data**.
- $\tau_{\mathbf{g}^* \times \hat{\mathbf{g}}}$: to compare the approaches using **future data** and our proposal (**prediction data**).

Numerical experiments

1. simulations with synthetically generated data;
2. an example with actual economic-financial time-series data.

Numerical experiments - real data

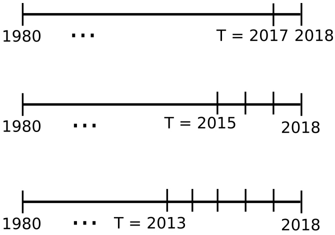
Example with actual economic-financial time-series data taked from the International Monetary Fund (IMF): https://www.imf.org/en/Data.

Numerical experiments - real data

- **Four alternatives**: Germany (1), Japan (2), Netherlands (3), and USA (4).
- **Three criteria**: gross national savings, inflation, and unemployment rate.
- **Time-series**: $t = 1980, \cdots, 2018$. (40 years).

Numerical experiments - real data

- We assume that the prediction must be done for the year 2018, which allows us to obtain the ranking $\mathbf{g}^*$ to compute the $\tau_{\mathbf{g}^c \times \mathbf{g}^*}$ and $\tau_{\mathbf{g}^* \times \hat{\mathbf{g}}}$;
- In RLS algorithm we consider different steps ($\lambda = 1, 3, 5$) — $T = 2017$, $T = 2015$ and $T = 2013$, respectively:



Numerical experiments - real data

Table: $\tau_{\mathbf{g}^c \times \mathbf{g}^*}$ and $\tau_{\mathbf{g}^* \times \hat{\mathbf{g}}}$ values for different T and λ.

	$T = 2017,\ \lambda = 1$	$T = 2015,\ \lambda = 3$	$T = 2013,\ \lambda = 5$
$\tau_{\mathbf{g}^c \times \mathbf{g}^*}$	0.50	0.33	0.50
$\tau_{\mathbf{g}^* \times \hat{\mathbf{g}}}$	0.00	0.00	0.50

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Conclusion

- To use of a prediction strategy has the potential to provide rankings that are closer to the ranking provided by future (with unobserved data).
- There is a relevant distance between the ranking provided by the approaches based on current and future data.
- Even for a small number of samples, the proposed method has presented good performance.

Thank you

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