**Abstract**

This work generalizes an existing framework of nonseparable oversampled lapped transforms (NSOLTs) to effectively represent complex-valued images.

- NSOLT is a redundant transform with linear-phase, compact-supported and perfect-reconstruction property.
- Generalized structure of NSOLT is proposed to cover complex-valued atomic images.
- Effectiveness of the structure is verified by evaluating the sparse approximation performance.

**Key words** - Complex dictionary, NSOLT, Parseval tight frame, Millimeter wave radar, Sparse approximation

**Introduction**

- Sparsity-based image restoration
  - denoising, deblurring, super-resolution, inpainting, compressive sensing, etc.
- NSOLT [1]
  - Effective for image restoration problem
  - Example-based design (dictionary learning)
  - Problem
  - Existing NSOLT can only deal with real-valued signal.
  - Insufficient degrees of freedom for complex-valued images.
- Objective
  - Generalize existing real-valued NSOLT (RNSOLT) to complex number field.
  - Evaluate the effectiveness by sparse approximation performance of complex-valued images.

**Complex Nonseparable Oversampled Lapped Transform**

- Complex-valued redundant transform satisfying linear-phase, compact-supported and perfect-reconstruction property.
- Type-I CNSOLT includes Type-I RNSOLT as a subset.
- Lattice structure of analysis bank is obtained by a cascade construction as $\mathbf{E}(\mathbf{z}) = \Phi \left[ \mathbf{K}_0 \mathbf{G}_0^{(a)}(\mathbf{z}_0) \right] \left[ \mathbf{K}_1 \mathbf{G}_1^{(b)}(\mathbf{z}_1) \right] \mathbf{E}_0(\mathbf{z})$,

**Performance Evaluation**

- Design example
  - RNSOLT and CNSOLT

**Sparse Approximation**

- Problem settings
  - Find a sparse vector $\mathbf{y}$ that minimize:
    $$ \| \mathbf{x} - \mathbf{D}\mathbf{y} \|_0 \leq K, $$ (1)
  - $K$ : Number of nonzero elements
- Iterative hard thresholding (IHT) algorithm
  - Heuristic algorithm for solving (1)
    $$ \mathbf{y}^{(i)} \leftarrow H_k(\mathbf{y}^{(i-1)}) + \mathbf{D}^T [\mathbf{x} - \mathbf{D}\mathbf{y}^{(i-1)}] $$ (2)
- $H_k(\cdot)$ : Hard thresholding operator.

**Results**

- Sparse approximate simulation
  - Original image $\mathbf{x}$: millimeter wave radar complex image in Fig. 4.
  - The intensity and hue show the magnitude and phase characteristics, respectively.

**Conclusion**

- Proposed the lattice structure of 2-D CNSOLT
- Employed CNSOLT as a sparse modeling method for complex-valued images
- Effectiveness of the proposed method for millimeter wave radar images was examined by sparse approximation performance.

**References**


**Acknowledgements**

This work was supported by JSPS KAKENHI Grant Numbers JP26420347, JP17H03261.