Let the gradient of the objective function in (6) with respect to $y$ equal zero,

$$\nabla \mathcal{L}(\hat{y}) = 0,$$

which results in

$$y = (A^T Q A)^{-1} A^T Q d.$$

Substituting (9) into (6) yields

$$\begin{align*}
\min_{\hat{y}} (1/2) d^T (Q^T Q) d & \quad \text{s.t. } H \hat{y} = H y \\
\text{where } H &= A (A^T Q A)^{-1} A^T.
\end{align*}$$

By letting $D = d^T$, (9a) can be rewritten as

$$(12) \quad \begin{align*}
Q Z &= (Q^T Q) Z \quad \text{s.t. } \quad H Z = \Delta m.
\end{align*}$$

Further, by letting $m_i = z_i$ from (6b), we have

$$(13a) \quad \begin{align*}
D_x &= y_i - x_i^* = \gamma_i - \beta_i A_i^T z_i, \quad 1 \leq i \leq M.
\end{align*}$$

An SDP-based localization algorithm is finally expressed as

$$(13b) \quad \begin{align*}
\min \{ \| \Theta - \Theta_{\text{true}} \|^2 \} & \quad \text{s.t. } \quad \Theta = \{ \Theta_{\text{true}}, \Theta_{\text{true}} \}.
\end{align*}$$

3 Localization Algorithm With Position Uncertainties

When the locations of the anchor nodes are not precise, which is mostly the case in practice, the sensor positions with errors can be expressed as $x_i = \hat{x}_i + \delta_i, \quad 1 \leq i \leq M$. This leads to time differences $t_{ij} = (s_i - s_j)/(c_n u)$, from (9b), we have

$$D_{ij} = s_i - s_j = (\hat{x}_i - \hat{x}_j)/(c_n u).$$

The next step would be to calculate $y_i$, i.e., (6) using the value of $\hat{u}$. The clock skew is estimated from $y_i$ as

$$\hat{\tau} = \frac{1}{\hat{y}_i}.$$