Tensor Dictionary Learning with representation quantization for Remote Sensing Observation Compression

Anastasia Aidini\textsuperscript{1,2}, Grigoris Tsagkatakis\textsuperscript{1}, Panagiotis Tsakalides\textsuperscript{1,2}

\{aidini, greg, tsakalid\}@ics.forth.gr

\textsuperscript{1}Institute of Computer Science  FORTH, Greece

\textsuperscript{2}Computer Science Department, UOC

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Motivation

Remotely sensed images are used for:
- forest monitoring
- disaster evaluation
- land cover estimation

Challenges:
- Increasing spatial, spectral and temporal resolutions of the images
- Increasing storage and transmission requirements
- High dimensional observations modeled as tensors
- High spacial, spectral and temporal redundancies
Problem

Compression of High-Dimensional Remote Sensing Observations that
- Includes quantization and coding
- Achieves high compression ratio
- Retains the structure of the data
- Can handle arbitrary high dimensional data structures

Proposed solution:
A novel tensor dictionary learning method is used to compress every new sample as a vector of sparse coefficients corresponding to the elements of the learned tensor dictionary, given a set of previous samples.
Related Work

- 3D Wavelet transform on the full data cube.
- A combination of JPEG2000 with Discrete Wavelet Transform or Principal Components Analysis for spectral decorrelation.
- Tensor-based approaches using tensor decompositions by transmitting all the factors of the decomposition.
A tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is a $N$-way array, a higher-order generalization of vectors and matrices.

The mode-$n$ unfolded matrix $X_n \in \mathbb{R}^{I_n \times \prod_{i \neq n} I_i}$ corresponds to a matrix with columns being the vectors obtained by fixing all indices of $\mathcal{X}$ except the $n$-th index.
Tensor Rank

The outer product of \( N \) vectors yields a rank-1 \( N \)-way tensor.

Every tensor can be written as a sum of rank-1 tensors

\[
\mathcal{X} \approx \sum_{r=1}^{R} a_r^{(1)} \circ a_r^{(2)} \circ ... \circ a_r^{(N)}
\]

- The *rank* of a \( N \)-way tensor \( \mathcal{X} \) is the smallest number \( R \) of rank-1 tensors needed to synthesize \( \mathcal{X} \).
- No straightforward algorithm to determine the rank of a specific given tensor (NP-hard problem).
CP Decomposition

CANDECOMP/PARAFAC (CP) decomposition represents a $N$-order tensor $\mathbf{X} \in \mathbb{R}^{I_1 \times \ldots \times I_N}$ as a linear combination of rank-1 tensors in the form

$$
\mathbf{X} = \sum_{r=1}^{R} \lambda_r \mathbf{a}_r^{(1)} \circ \mathbf{a}_r^{(2)} \circ \ldots \circ \mathbf{a}_r^{(N)} = \mathbf{D} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \times_3 \ldots \times_N \mathbf{A}^{(N)}
$$

where $\mathbf{A}^{(n)} = [a_1^{(n)} \ a_2^{(n)} \ \ldots \ \ a_R^{(n)}]$, $n = 1, \ldots, N$ are the factor matrices and $\mathbf{D} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_R) \in \mathbb{R}^{R \times \ldots \times R}$ is a diagonal core tensor.

![Diagram showing tensor decomposition](image-url)
Training Process

1. Learn a tensor dictionary $\mathcal{D} \in \mathbb{R}^{I_1 \times \cdots \times I_N \times K}$ of $K$ rank-1 tensors $\mathcal{D}(k) \in \mathbb{R}^{I_1 \times \cdots \times I_N}$, $k = 1, \ldots, K$, by minimizing

$$
\min_{\mathcal{D}, \mathbf{A}} \frac{1}{2} \| \mathbf{X} - \mathcal{D} \times_{N+1} \mathbf{A} \|_F^2
$$

subject to $\| \mathbf{A}(t,:) \|_0 \leq \lambda$, $\forall t = 1, \ldots, T$,

where $\mathbf{X} = (\mathbf{X}^1, \mathbf{X}^2, \ldots, \mathbf{X}^T) \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N \times T}$ is a set of $T$ training samples and $\mathbf{A} \in \mathbb{R}^{T \times K}$ contains the corresponding sparse coefficients.

2. Learn a coding dictionary which maps a binary number to a set of $2^{bit}$ symbols by equally partitioning the range of values of the coefficient matrix $\mathbf{A}$. 
Proposed Tensor Dictionary Learning Method

Introducing the auxiliary variable $\mathbf{G} \in \mathbb{R}^{T \times K}$, we apply the Alternating Direction Method of Multipliers to solve the reformulated problem

$$\min_{\mathbf{D}, \mathbf{A}, \mathbf{G}} \frac{1}{2} \| \mathcal{X} - \mathcal{D} \times_{N+1} \mathbf{A} \|_F^2 + \lambda \sum_{t=1}^{T} \| \mathbf{G}(t, :) \|_0$$

subject to $\mathbf{G} = \mathbf{A}$

The augmented Lagrangian function is given by

$$\mathcal{L}(\mathbf{A}, \mathbf{G}, \mathbf{D}, \mathbf{Y}) = \frac{1}{2} \| \mathcal{X} - \mathcal{D} \times_{N+1} \mathbf{A} \|_F^2 + \lambda \sum_{t=1}^{T} \| \mathbf{G}(t, :) \|_0 + < \mathbf{Y}, \mathbf{G} - \mathbf{A} > + \frac{p}{2} \| \mathbf{G} - \mathbf{A} \|_F^2,$$

where $\mathbf{Y} \in \mathbb{R}^{T \times K}$ is the Lagrange multiplier matrix and $p > 0$ denotes the step size parameter.
Tensor Dictionary Learning Algorithm

We solve the problem iteratively by minimizing $L$ with respect to each variable while keeping the others fixed. At each iteration $l$ we update:

- **Sparse coefficient matrix $A$:**
  \[
  \nabla_A L = 0 \implies A \leftarrow (X_{(N+1)} \cdot D_{(N+1)}^T + Y + p \cdot G) \cdot (D_{(N+1)} \cdot D_{(N+1)}^T + p \cdot I)^{-1}
  \]

- **Auxiliary variable $G$:**
  \[
  \nabla_G L = 0 \implies G \leftarrow H_\lambda(A - \frac{Y}{p}), \text{ where } H_\lambda(x) = \begin{cases} x, & |x| > \lambda \\ 0, & \text{otherwise} \end{cases}
  \]

- **Tensor dictionary $D$:**
  \[
  \nabla_D L = 0 \implies D^{(l)} \leftarrow D^{(l-1)} + X \times_{N+1} A^{-1} \text{ and normalization}
  \]

- **Lagrangian multiplier matrix $Y$:**
  \[
  Y^{(l)} \leftarrow Y^{(l-1)} + p \cdot (G - A), \text{ where } p = 0.6 \text{ in our setup}
  \]
Compress each new sample $\hat{X} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$ as a sparse vector of coefficients $a \in \mathbb{R}^K$ such that

$$\hat{X} = D \times_{N+1} a,$$

where $\|a\|_0 \leq \lambda$.

Quantize $a$ to $b$ bits using a uniform quantizer $Q$.

Encode $Q(a)$ using Huffman coding and the learned encoding dictionary.

$Q \rightarrow 10011011$
Decompression

1. Decode the transmitted coefficients $a_q = Q(a)$ using the learned Huffman dictionary.
2. Decompress the sample as $\hat{X} \approx D \times_{N+1} a_q$. 
Experiments

- Data: Time series of satellite derived observations of normalized difference vegetation index (NDVI).

- Sample size: $200 \times 200 \times 7$
  The last dimension indicates the number of days.

- Training samples: 50

- The recovery performance is measured in terms of the Normalized Mean Square Error (NMSE) which is defined as $\text{NMSE} = \frac{\|Y - \hat{Y}\|_2^2}{\|Y\|_2^2}$. 
**Number of Atoms of the Dictionary**

**Figure:** Reconstruction quality for each test sample and different number of atoms of the dictionary, using 80% sparsity level and 8 bits of quantization.
**Figure:** Reconstruction quality for each test sample and different sparsity levels, using a dictionary with 500 atoms and 8 bits of quantization.
Number of Quantization Bits

**Table:** Reconstruction error for different samples as a function of quantization bit number.

<table>
<thead>
<tr>
<th>Number of Bits</th>
<th>1st Sample</th>
<th>10th Sample</th>
<th>25th Sample</th>
<th>35th Sample</th>
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<tr>
<td>4</td>
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<td>0.3226</td>
<td>0.3510</td>
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<td>0.2044</td>
<td>0.2425</td>
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</table>
Experimental Results

Comparison with State-of-the-art Compression Algorithm

Figure: Reconstruction quality for several test samples, using 0.06 bpppb.

Table: Reconstruction quality for different number of bpppb.

<table>
<thead>
<tr>
<th>NMSE</th>
<th>bpppb</th>
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<tr>
<td></td>
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<td>Our Method</td>
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<tr>
<td>JPEG2000</td>
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</tbody>
</table>
Conclusion

- An end-to-end compression algorithm is proposed that includes quantization and coding.

- A novel tensor dictionary learning method based on CP decomposition is presented for compression purposes.

- The proposed scheme can handle arbitrary high dimensions.

- Our method is evaluated on 3D remote sensing observations.
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