A Low-Complexity LS Turbo Channel Estimation Technique for MU-MIMO Systems

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Summary

• Turbo Receiver improves the MUI problem in MU-MIMO sys.
• However, the complexity expands as Num. of ANT increases
• Because, in order to deal Spatial matrix $\Gamma$ correctly

NEW: Low complexity LS algo. for Turbo receiver
• Independent of Rx Ants.: $O(\kappa^3 N_T^3) \leftarrow O(\kappa^3 N_T^3 N_R^3)$
• No Accuracy Deterioration !
• Algebraic property of Cov. Matrix $\mathbf{R}_{XX}$ is utilized
System Model

The $u$-th Transmitter

$\begin{align*}
    b_u(i) & \xrightarrow{\text{Binary information}} c_u(i_c) \\
    & \xrightarrow{\text{CC}} c_u(\Pi_u(i_c)) \\
    & \xrightarrow{\Pi_u} \mathbf{x}_{t,k'}(l) \\
    & \xrightarrow{\text{Serial-to-parallel convertor}} k' = 1
\end{align*}$

Receiver

$\begin{align*}
    \mathbf{y} & \xrightarrow{\text{Power}} \mathbf{p} \\
    & \xrightarrow{\Pi_u^{-1}} \mathbf{x}_u \\
    & \xrightarrow{\text{deinterleaver}} \mathbf{\hat{x}}_d \\
    & \xrightarrow{\text{Soft Replica}} \mathbf{\hat{x}}_{d,k}
\end{align*}$

The $u$-th Decoding Process

The $U$-th Decoding Process

TX Data Format

$\mathbf{X} = [\mathbf{X}_t \quad \mathbf{X}_d]$
LS Channel Estimation

$$\hat{H} = \arg\min_H \mathcal{L}_t(H)$$

$$= Y_t X_t^+ = Y_t X_t^H (X_t X_t^H)^{-1}$$

$$\mathcal{L}_t(H) = \frac{1}{\sigma^2_Z} \|Y_t - HX_t\|^2$$

Rx Signal: $$Y = HX + Z$$

H: Channel
X: TX Signal
Z: AWGN $\sim \mathcal{C}\mathcal{N}(0, \sigma^2_Z)$
LS Turbo
Channel Estimation

\( \hat{H} = \arg\min_H \mathcal{L}_{td}(H) \), where \( \mathcal{L}_{td}(H) = \mathcal{L}_t(H) + \mathcal{L}_d(H) \)

\[
\mathcal{L}_d(H) = \frac{1}{\sigma_Z^2} \| Y_t - H \hat{X}_d \|_F^2
\]

\[
= \frac{1}{\sigma_Z^2} \text{tr} \left\{ (Y_d - H \hat{X}_d)^H \Gamma (Y_d - H \hat{X}_d) \right\}
\]

\[
\Gamma = (I_{NR} + \sum_u \frac{\Delta \sigma_d^2}{\sigma_Z^2} R_{H,u})
\]
**LS Solution:**

\[
\text{vec}\{\hat{H}\} = R^{-1}_{XX} \cdot \text{vec}\{R_{YX}\}
\]

**Gaussian Elimination?**

\[O((WUN_T N_R)^3)\]

**WUN_T N_R x WUN_T N_R Matrix**

\[
R_{XX} = R_{XX_t}^T \otimes I_{N_R} + \hat{R}_{XX_d}^T \otimes \Gamma
\]

\[
R_{YX} = R_{YX_t} + \Gamma \hat{R}_{YX_d}
\]

- \(X_t: WUN_T \times L_t\)
- \(\hat{X}_d: WUN_T \times L_d\)
- \(Y_t: N_R \times L_t\)
- \(Y_d: N_R \times L_d\)

\[O((WUN_T)^2 L_{td})\]

\[O(N_R^2 L_{td})\]

**System Size**
- \(W: CIR\) length
- \(U: \text{Num. of Users}\)
- \(N_T, N_R: \text{Tx, Rx Ants.}\)
- \(L_t, L_d: \text{TS, Data Len.}\)
\[ R_{XX} = R^{T}_{XX_t} \otimes I_{NR} + \hat{R}^{T}_{XX_d} \otimes \Gamma \]

\[ = \left( R^{T/2}_{XX_t} \otimes I_{NR} \right) J \left( R^{T/2}_{XX_t} \otimes I_{NR} \right)^{H} \]

\[ J = I_{WUNT} \otimes I_{NR} + R^{-H/2}_{XX_t} \hat{R}^{T}_{XX_d} R^{-1/2}_{XX_t} \otimes \Gamma \]

\[ \sim \mathcal{O}(WUNTNR) \]

\[ R_{XX}^{-1} = (\tilde{U}_Q \otimes U_{\Gamma}) \Sigma_{J}^{-1} (\tilde{U}_Q \otimes U_{\Gamma})^{H} \]

Unitary  Diagonal  Unitary

SVD:
\[ \mathcal{O}(N_{R}^{3}) \]

\[ Q = U_{Q} \Sigma_{Q} U_{Q}^{H} \]
\[ \Gamma = U_{\Gamma} \Sigma_{\Gamma} U_{\Gamma}^{H} \]

\[ \Sigma_{J} = I_{WUNTNR} + \Sigma_{Q} \otimes \Sigma_{\Gamma} \]
\[ \tilde{U}_{Q} = R_{XX_t}^{*} U_{Q} \]
\[ \text{vec}\{\hat{H}\} = \mathcal{R}_{xx}^{-1} \cdot \text{vec}\{R_{yx}\} \]

\[ \mathcal{R}_{xx}^{-1} = (\tilde{U}_Q \otimes U_\Gamma) \cdot \Sigma_j^{-1} \cdot (\tilde{U}_Q \otimes U_\Gamma)^H \]

\[ \hat{H} = \text{mat}_N[R] [\text{vec}\{\hat{H}\}] \]

\[ = U_\Gamma \text{mat}_N[v] \tilde{U}_Q^T \]

\[ v = \text{diag}\{\Sigma_j^{-1}\} \odot \text{vec}\{\tilde{U}_Q^H R_{yx} U_\Gamma^*\} \]

Consequently, \( O((WUN_T)^3) \) when \( WUN_T \ll N_R \)

\[ \leftarrow O\left( (WUN_T)^3 + N_R^3 + (WUN_T)^2 L_{td} \right) \]
Numerical Results

No Performance Degradation

Channel models: {PB 3km/h, VA 30km/h}
2Users, 4 x 12 MIMO, SNR=18dB

Complexity is Independent of Rx antennas

\[ O(\kappa^3 N_T^3 N_R^3) \]

System size: \( \kappa = WU \)

\[ O(\kappa^3 N_T^3) \]

Approx. \( \hat{R}_{H,u} \approx I_{NR} \)