Dynamic ADMM For Real-Time Optimal Power Flow

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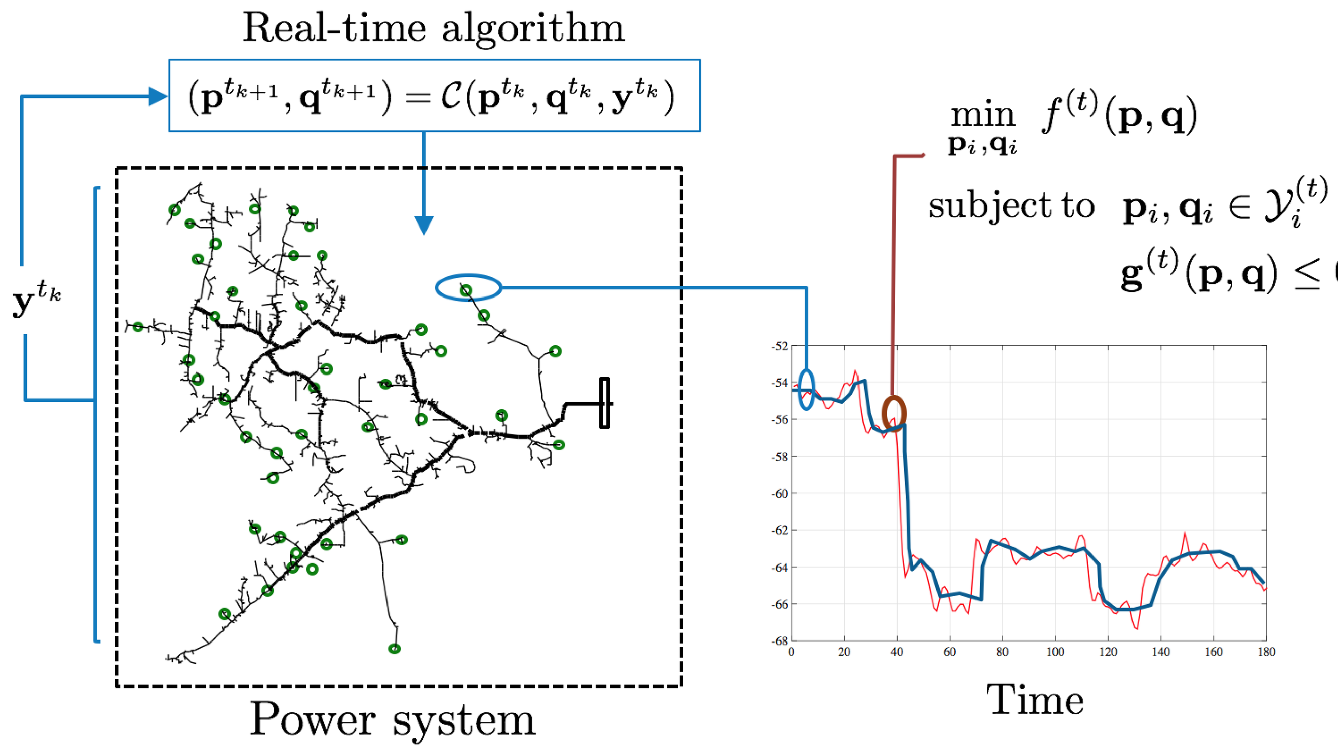
Objectives



Main Challenge:

1. For real-time implementations, it might be infeasible to solve OPF problem to convergence at a fast time scale
2. Solving OPF problem requires collecting measurements of the noncontrollable loads at all locations in real time

Contribution:

(a) Design a *feedback-based dynamic ADMM* that copes with approximation errors and avoides ubiquitous monitoring

(b) Leverage a linearizations of the AC power-flow equations [Dhople et al. 2015] to formulate convex surrogates of original OPF problem

(c) Prove the tracking ability of dynamic ADMM

Linearized Model

- **Approximate Linear Relationship** between voltage magnitudes and net injected power:

$$|V_n^k| \approx \sum_{i \in \mathcal{N}_D} [r_{n,i}^k (P_i - P_{l,i}^k) + b_{n,i}^k (Q_i - Q_{l,i}^k)] + \bar{a}_n^k$$

$$\bar{a}_n^k := a_n^k - \sum_{i \in \mathcal{N}} (r_{n,i}^k P_{l,i}^k + b_{n,i}^k Q_{l,i}^k).$$

- **Linearzied Version:**

$$\min_{\mathbf{p},\mathbf{q}} \sum_{i \in \mathcal{N}_D} f_i^k(\boldsymbol{u}_i) \qquad \text{(P1)}$$

$$\text{s.t.} \quad g_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) \leq 0, \forall n \in \mathcal{M},$$

$$\bar{g}_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) \leq 0, \forall n \in \mathcal{M},$$

$$\boldsymbol{u}_i \in \mathcal{Y}_i^k,$$

$$g_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) = V^{\min} - \bar{a}_n^k - \sum_{i \in \mathcal{N}_D} (r_{n,i}^k P_i + b_{n,i}^k Q_i) \leq 0,$$

$$\bar{g}_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) = \bar{a}_n^k + \sum_{i \in \mathcal{N}_D} (r_{n,i}^k P_i + b_{n,i}^k Q_i) - V^{\max} \leq 0.$$

Reformulation

Relaxed Version:

$$\min_{\mathbf{p},\mathbf{q},\mathbf{z},\mathbf{y}} \epsilon(\|\boldsymbol{z}\|^2 + \|\boldsymbol{y}\|^2) + \gamma(h(\boldsymbol{z}) + h(\boldsymbol{y})) + \sum_{i \in \mathcal{N}_D} f_i^k(\boldsymbol{u}_i) \qquad \text{(P2)}$$

$$\text{s.t.} \quad g_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) + z_n = 0, \forall n \in \mathcal{M},$$

$$\bar{g}_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) + y_n = 0, \forall n \in \mathcal{M},$$

$$\boldsymbol{u}_i \in \mathcal{Y}_i^k, \forall i \in \mathcal{N}_D.$$

Feedback-based Dynamic ADMM

- **Augmented Lagrangian (compact form):**

$$\mathcal{L}(\mathbf{u}, \mathbf{z}, \mathbf{y}; \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{i \in \mathcal{N}_D} f_i^k(\boldsymbol{u}_i) + \epsilon \sum_{n \in \mathcal{M}} (\|z_n\|^2 + \|y_n\|^2)$$
$$+ \gamma \sum_{n \in \mathcal{M}} (h(z_n) + h(y_n)) + \frac{\rho}{2} \sum_{n \in \mathcal{M}} (g_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) + z_n + \frac{\lambda_n}{\rho})^2$$
$$+ \frac{\rho}{2} \sum_{n \in \mathcal{M}} (\bar{g}_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) + y_n + \frac{\mu_n}{\rho})^2.$$

- **Incorporate System Feedback:**

$$g_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) \leftrightarrow V^{\min} - |\hat{V}_n^k|,$$

$$\bar{g}_n^k(\{\mathbf{u}_i\}_{i \in \mathcal{N}_D}) \leftrightarrow |\hat{V}_n^k| - V^{\max},$$

here we use the system feedback $|\hat{V}_n^k|$ instead of $|V_n^k|$

- **Projected-gradient Step:**

$$\begin{pmatrix} P_i^k \\ Q_i^k \end{pmatrix} = \text{proj}_{\mathcal{Y}_i^k} \begin{pmatrix} P_i^{k-1} - \alpha \frac{\partial \mathcal{L}}{\partial P_i}|_{P_i^{k-1}, Q_i^{k-1}} \\ Q_i^{k-1} - \alpha \frac{\partial \mathcal{L}}{\partial Q_i}|_{P_i^{k-1}, Q_i^{k-1}} \end{pmatrix}.$$

Dynamic ADMM:

(S1) For each $i \in \mathcal{N}_D$, update P_i^k, Q_i^k via gradient projection based on the most up-to-date measurements.

(S2) Collect voltage measurements $|\hat{V}_n^k|, n \in \mathcal{M}$.

(S3) For each $n \in \mathcal{M}$, update auxiliary variables z_n^k, y_n^k.

(S4) For each $n \in \mathcal{M}$, update dual variables λ_n^k, μ_n^k as

$$\lambda_n^k = \lambda_n^{k-1} + \rho(V^{\min} - |\hat{V}_n^k| + z_n^k), n \in \mathcal{M},$$

$$\mu_n^k = \mu_n^{k-1} + \rho(|\hat{V}_n^k| - V^{\max} + y_n^k), n \in \mathcal{M}.$$

Extension to Nonsmooth Objective:

- Unlike existing state of art methods like double smoothing [Dall'Anese et al. 2016][Simonetto et al. 2014], our method can handle nonsmooth objectives such as ℓ_1 norm and the convergence analysis is still valid
- Change projected-gradient step to a proximal-gradient step with soft-threshold operator

Convergence Analysis

Bounded Variation Assumption: temporal variability of optimal solution of (P2) is bounded with constants $\sigma_u, \sigma_\eta, \sigma_\psi$

Bounded Difference Assumption: $\left| |V_n^k| - |\hat{V}_n^k| \right| \leq e$

Main Theorem: Define $\boldsymbol{w} := (\boldsymbol{\eta}; \boldsymbol{u}; \boldsymbol{\psi})$ and let $\{\boldsymbol{w}^k\}$ be the sequence generated by dynamic ADMM. Further, let $\boldsymbol{w}^{*,k}$ be an optimal solution of (P2) at time k. Then,

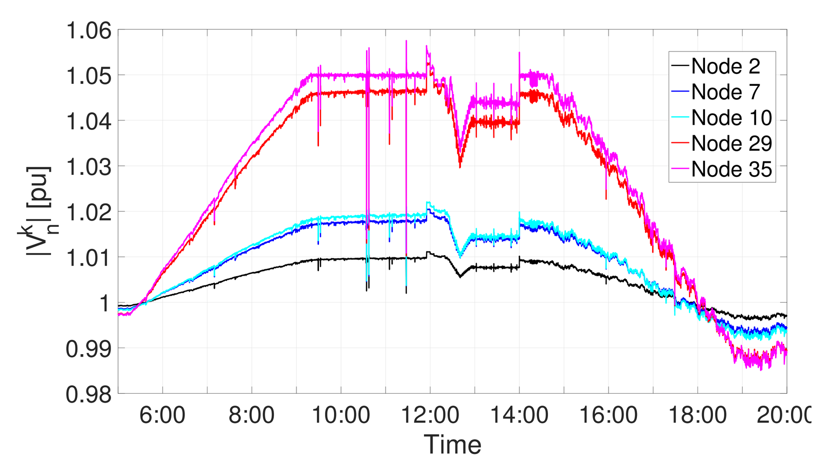
$$\lim_{k \to \infty} \sup \|\boldsymbol{w}^k - \boldsymbol{w}^{*,k}\|_2 = \frac{\alpha(e)}{1 - r} + \frac{r\beta(\sigma)}{1 - r},$$

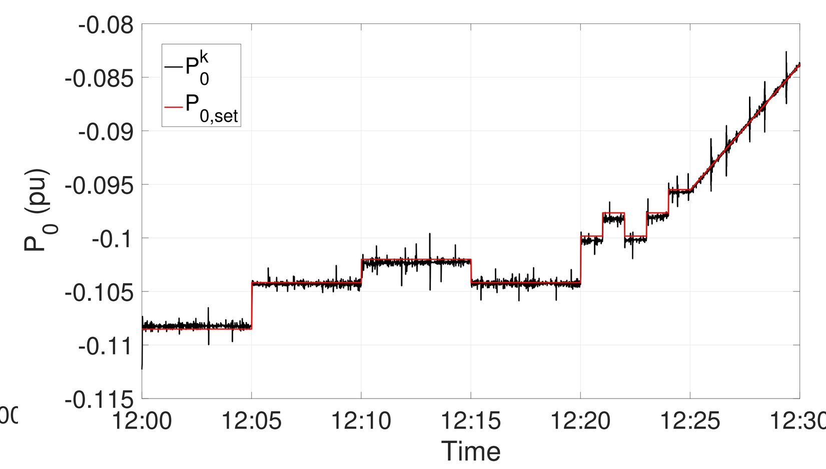
where $0 < r < 1$, $\alpha(e) = \frac{\rho e}{\epsilon} + 4\rho e + \alpha\rho e \sum_{i,n}(r_{n,i} + b_{n,i})$ and $\beta(\sigma) = \sigma_u + \sigma_\eta + \sigma_\psi$.

Key Idea: We prove that iterates produced by dynamic ADMM converge to a *neiborhood of the optimal solution* of (P2)

Numerical Experiments

- The optimization objective is chosen to be $f_i(P_i, Q_i) = c_p(P_{\text{av},i} - P_i)^2 + c_q(Q_i)^2 + \bar{c}_q|Q_i|$, and $c_p = 3, c_q = 1, \bar{c}_q = 0.1$
- The voltage limits are set to be $V^{\min} = 0.95$pu, $V^{\max} = 1.05$pu
- The generation profiles are simulated based on real solar irradiance data and have a granularity of 1 second
- First figure shows that voltage regulation is enforced in real time and a flat voltage profile is obtained
- Second figure shows that algorithm is able to track power setpoints at the point of coupling in real time





- The following figure compares dynamic ADMM and double smoothing [Dall'Anese et al. 2016], our framework is able to achieve smaller voltage violation