ENERGY BLOWUP FOR TRUNCATED STEABLE LTI SYSTEMS

Introduction

Signal Reconstruction

System approximation process 1:
The time variable \( t \in \mathbb{R} \) is in the argument of \( h \).

\[ (T_1 f)(t) = \sum_{k=-\infty}^{\infty} f(k) h(t-k), \quad t \in \mathbb{R} \]

For all \( f \in \mathcal{PW}_2 \) and all stable LTI systems \( T \): \( \mathcal{PW}_2 \to \mathcal{PW}_2 \) we have:

- norm convergence:
  \[ \lim_{N \to \infty} \int_{-\infty}^{\infty} |(Tf)(t) - (T_1 f)(t)|^2 \, dt = 0 \]

- uniform convergence:
  \[ \lim_{N \to \infty} \max_{\mathbb{R}} |(Tf)(t) - (T_1 f)(t)| = 0. \]

The system approximation process \( T_1 f \) converges in the \( \mathcal{PW}_2 \)-norm and uniformly on the real axis.

System approximation process 2:
The time variable \( t \in \mathbb{R} \) is in the argument of \( f \).

\[ (T_2 f)(t) = \sum_{k=-\infty}^{\infty} f(k) h(t-k) \]

For all \( f \in \mathcal{PW}_2 \) and all stable LTI systems \( T \): \( \mathcal{PW}_2 \to \mathcal{PW}_2 \) we have:

uniform convergence:
\[ \lim_{N \to \infty} \max_{\mathbb{R}} |(Tf)(t) - (T_2 f)(t)| = 0. \]

\( T_N^1 f \) and \( T_N^2 f \) have the same global convergence behavior.

Notation

Paley–Wiener space \( \mathcal{PW}_2 \): Space of signals \( f \) with a representation \( f(z) = 1/(2\pi) \int_{-\pi}^{\pi} f(u) e^{iuz} \, du \), \( z \in \mathbb{C} \), for some \( f \in L^2[-\pi, \pi] \). Norm:
\[ ||f||_{\mathcal{PW}_2} = \left( \int_{-\pi}^{\pi} |f(u)|^2 \, du \right)^{1/2}. \]

Stable LTI systems: A linear system \( T \) is called stable linear time invariant (LTI) system if:
- \( T \) is bounded, i.e., \( ||T|| \leq \sup_{f \in \mathcal{PW}_2} ||Tf||_{\mathcal{PW}_2} < \infty \) and
- \( T \) is time invariant, i.e., \( (T(t-\cdot))(t) = (T(t))(t) \) for all \( f \in \mathcal{PW}_2 \) and \( t, \alpha \in \mathbb{R} \).

Representation: For every stable LTI system \( T \) : \( \mathcal{PW}_2 \to \mathcal{PW}_2 \) there exists exactly one function \( h_T \in L^1[-\pi, \pi] \) such that \( |(Tf)(t)| = \frac{1}{2\pi} \int_{-\pi}^{\pi} |h_T(u) \tilde{f}(u)| e^{iut} \, du \) for all \( f \in \mathcal{PW}_2 \). We have \( h_T = T(1) \).