Introduction

• Sequential detectors can significantly reduce the average number of samples compared to fixed sample size tests with the same reliability.
• Sequential detectors for multiple hypotheses can solve non-binary decision problems.
• Fully distributed methods exploit the inherent scalability, fault-tolerance, and absence of a single point of failure in sensor networks.
• Distributional uncertainties in real-world applications call for robust solutions.

Contributions

• We propose the Consensus + Innovations Matrix Sequential Probability Ratio Test (C+IRMSPRT) as a sequential multiple hypothesis test for distributed sensor networks.
• We provide an accurate prediction of the average stopping time of the C+IRMSPRT.
• We robustify the C+IRMSPRT using least favorable densities (LFDs).
• We validate the performance of the C+IRMSPRT and the robust LFD-C+IRMSPRT in a shift-in-variance test.
• We analyze the impact of network size and connectivity on the performance.

Problem Formulation

• Detect the presence of one out of M signals εm(t) with different variances σm2 in a non-Gaussian environment with a distributed sensor network.
• Shift-in-variance test between M hypotheses under ε-contamination:
  \[ H_m: \epsilon_m(t) \sim \mathcal{N}(0, \sigma_m^2) \]
  \[ p_{\text{true}} = (1 - \epsilon) \sigma_h^2 + \epsilon \sigma_m^2 \]
  \[ p_{\text{true}}, p_{\text{nom}}, p_{\text{cont}}: \text{true, nominal and contaminated probability density functions under } H_m \]

The Consensus + Innovations Matrix SPRT (C+IRMSPRT)

• Calculation of the log-likelihood ratio of node k at time instant t and the corresponding test statistic for the hypothesis pair Hm, Hn:
  \[ q_{mn}(t) = \log \left( \frac{p_{mn}(t)}{p_{nm}(t)} \right) \]
  \[ S_{mn}(t) = \sum_{\ell=0}^{t} w_{\ell} q_{mn}(\ell) \]

Least Favorable Densities (LFDs)

• LFDs of Huber’s clipped likelihood ratio test for some ε_m, ε_n > 0 [4]
  \[ q_m = \max \{ cq_m^{\epsilon_m}, 1 - c \sigma_m^2 \} \]
  \[ q_n = \max \{ cq_n^{\epsilon_n}, 1 - c \sigma_n^2 \} \]

Robustifying the C+IRMSPRT

• We replace the log-likelihood ratio in (1) by the clipped log-likelihood ratios of the LFDs to obtain a robust test statistic:
  \[ q_{\text{clipped}}(t) = \log \left( \frac{p_{mn}(t)}{p_{nm}(t)} \right) \]

Results: Detecting the presence of one out of M signals

Setup

• four networks of different size and connectivity
  \[ M = 4 \text{ signals with variances } \sigma_n \in \{1, 2, 4, 16\} \]
• measurement noise: \( h_{\text{true}} = \mathcal{N}(0, 0.1) \), \( \epsilon \in [0, 0.3] \)
• probability of false alarm: \( \alpha_{\text{nom}} = 0.01 \)
• 1 000 Monte-Carlo runs

LFD-C+IRMSPRT

• accurate detection results up to 10% contamination (30% under \( H_0 \)) irrespective of network size and connectivity
• network connectivity impacts average stopping time

Related Work