An Efficient Active Set Algorithm for Covariance Based Joint Data and Activity Detection for Massive Random Access with Massive MIMO

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Motivation
- The uncoordinated random access is a challenging task in massive machine-type communication (mMTC).
- A large number of sporadically active devices wish to send small data to the base-station (BS) in the uplink.
- The BS acquires the active devices and their data by detecting the transmitted presigned nonorthogonal signature sequences.
- Covariance based approach [1, 2, 3]: formulate the detection problem as a maximum likelihood estimation (MLE) problem.
- The state-of-the-art coordinate descent (CD) algorithm doesn’t take advantage of the sparsity of the true solution.

Main Contribution
- Perform the covariance based approach for joint data and activity detection.
- Propose a computationally efficient active set algorithm with convergence guarantee.

System Model
- Single cell with one BS equipped with M antennas. 
- N single-antenna devices, K of which are active at a time. 
- Each active device wishes to transmit J bits of data to the BS.
- Each device n has a unique signature sequence set \( S_n = \{s_{n,k} \} \), where \( s_{n,k} \in \mathbb{C}^{Q \times 1}, 1 \leq k \leq Q \leq 2^L \), and \( L \) is the signature sequence length.
- Channel \( \sqrt{P_h} h_{n,k} \) of user n includes both large-scale fading component \( g_k \) and small-scale fading component \( h_{n,k} \) following the i.i.d. complex Gaussian distribution.
- Whether or not \( s_{n,k} \) is transmitted is indicated as \( \gamma_{n,k} \in \{0,1\} \), which satisfies \( \sum_k \gamma_{n,k} = 1 \) and \( \sum_k s_{n,k} \) indicates that device n is active.
- Define \( S = \{S_1, S_2, ..., S_N\} \) and \( \Gamma = \{\gamma_{1,1}, \gamma_{1,2}, ..., \gamma_{N,K}\} \).

Problem Formulation and Analysis
- Let \( f(\gamma) \) denote the objective function of problem (3). The gradient of \( f(\gamma) \) with respect to \( \gamma_{n,k} \) is
  \[
  \nabla f(\gamma)|_{\gamma_{n,k}} = \nabla \log |\Gamma| - s_{n,k}^H S \Gamma S_{n,k} \nabla \log |\Gamma| - s_{n,k}^H S \Gamma S_{n,k} \nabla \log |\Gamma| - s_{n,k}^H S \Gamma S_{n,k}
  \]
  The first-order (necessary) optimality condition of problem (3) is
  \[
  \nabla f(\gamma)|_{\gamma_{n,k}} \geq 0 \quad \text{if} \quad \gamma_{n,k} = 0, \quad \forall n \neq n_i \quad \text{and} \quad \gamma_{n_i} = 0, \quad \forall n_i \neq n_k.
  \]

Active Set Algorithm
- To fully exploit the sparsity of the true solution of (3), the active set should:
  - contain the indices of active sequences;
  - have the smallest possible cardinality.
- At the k-th iteration, the proposed selection strategy of the active set \( A^k \) is
  \[
  A^k = \left\{ (n,q) : \gamma_{n,q} > \varepsilon \text{ or } |\mathbf{V}(\gamma^{k-1})| - s_{n,q}^H S \Gamma S_{n,q} |\mathbf{V}(\gamma^{k-1})| - s_{n,q}^H S \Gamma S_{n,q} |\mathbf{V}(\gamma^{k-1})| - s_{n,q}^H S \Gamma S_{n,q} \right\},
  \]
  where \( \varepsilon > 0 \) and \( \mathbf{V}(\gamma^{k-1}) \) is the subvector of \( \gamma^{k-1} \) indexed by \( A^k \).

System Model (Cont.)
- The received signal \( Y \in \mathbb{C}^{M \times J} \) at the BS can be expressed as
  \[
  Y = \sum_{n=1}^{N} \sum_{k=1}^{Q} s_{n,k} \gamma_{n,k} \mathbf{x}_n \mathbf{h}_{n,k}^H + W
  \]
  where \( W \in \mathbb{C}^{M \times J} \) is the effective i.i.d. Gaussian noise with variance \( \sigma^2_W \).
- For given \( \gamma \) (diagonal entries of \( \Gamma \)), the m-th column of \( Y \) can be seen as independent samples from a complex Gaussian distribution as
  \[
  y_m \sim \mathcal{CN}(0, \sigma^2_W + \sigma^2_I),
  \]
  where \( \Lambda \) is a block diagonal matrix with each block being the all-one matrix \( E \in \mathbb{R}^{n \times n} \), and \( I \) is an identity matrix.
- Since there is at most one non-zero entry in each diagonal block \( D_n \in \mathbb{R}^{n \times n} \), the covariance matrix \( \Sigma = YY^H/M \) is computed by averaging over different antennas.
- The constraint \( \gamma \geq 0 \) is due to the fact that \( \gamma_{n,k} = (g_k h_{n,k})^2 \geq 0 \) for all n and q.

Algorithm 1: Proposed active set PG algorithm for solving problem (3)

1. Initialization: Create an empty set \( A^0 \), \( \gamma^0 \), \( \varepsilon^0 > 0 \) and \( \gamma^0 \); \( \Delta > 0 \).
2. Select the active set \( A^k \) according to (5).
3. Apply the spectral PG algorithm \( (4) \) to solve the subproblem (6) until (7) is satisfied.
4. Set \( k = k + 1 \), and go to step 2.
5. Output: \( \gamma^k \).