Efficiently and Globally Solving Joint Beamforming and Compression Problem in the Cooperative Cellular Network via Lagrangian Duality

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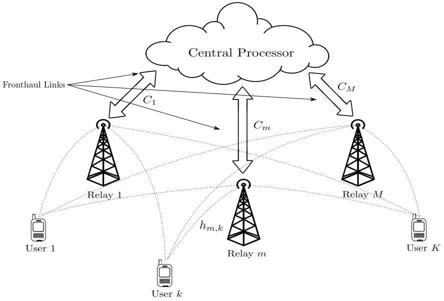
IEEE ICASSP, Singapore, May 7–13, 2022

Joint Beamforming and Compression Problem



- Cooperative cellular network
  - rate-limited fronthaul
  - effectively mitigating multiuser intercell interference
  - joint processing at CP
- Joint beamforming and compression problem

Literature Review

- Uplink $\Rightarrow$ well solved
- Downlink

Simeone13 Maximize the weighted sum-rate
$\Rightarrow$ stationary point [1]

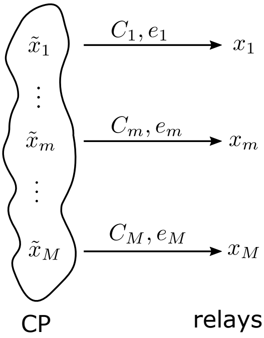
Liu21 Minimize the total power
$\Rightarrow$ duality results and global solution [2]

This paper Minimize the total power
$\Rightarrow$ global solution with high efficiency [3]

System Model

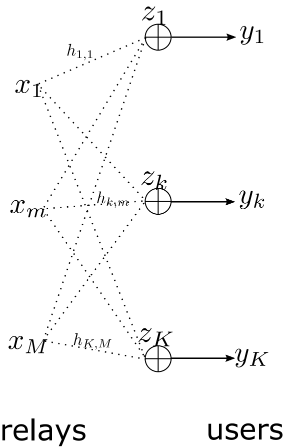
- A cooperative cellular network consists of
  - one CP,
  - M single-antenna relay-like BSs (will be called relays for short later),
  - K single-antenna users.
- Users and relays are connected by noisy wireless channels.
- Relays and the CP are connected by noiseless fronthaul links of finite rate.
- Let $\mathcal{M}$ and $\mathcal{K}$ denote the sets of the relays and the users, respectively.
- The channel between any users and relays is known at the CP.

Compression Model



- Transmitted signal at CP $\tilde{\boldsymbol{x}} = \sum_{k=1}^{K} \boldsymbol{v}_k s_k$, where
  - $\boldsymbol{v}_k = [v_{k,1}, \ldots, v_{k,M}]^{\mathsf{T}}$ is a beamforming vector,
  - $s_k \sim \mathcal{CN}(0, 1)$ is the information signal for user k
- Compressed before transmitted
- Compression error $\boldsymbol{e} = [e_1, \ldots, e_m]^{\mathsf{T}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q})$
- Covariance matrix $\mathbf{Q}$
- Received signal at relays $x_m = \sum_{k=1}^{K} v_{k,m} s_k + e_m$

Channel Model



- Received signal at users:
  $y_k = \sum_{m=1}^{M} h_{k,m} x_m + z_k$
- Transmitted signal at relays: x_m
- $h_{k,m}$ is the channel coefficient from relay m to user k, and
- $\{z_k\}$ are i.i.d. additive Gaussian noise distributed as $\mathcal{CN}(0, \sigma^2)$.

Total Transmit Power, SINR and Compression Rate

- Received signal at users with $\boldsymbol{h}_k = [h_{k,1}, \ldots, h_{k,M}]^\dagger$:

$$y_k = \boldsymbol{h}_k^\dagger \left(\sum_{i=1}^{K} \boldsymbol{v}_i s_i \right) + \boldsymbol{h}_k^\dagger \boldsymbol{e} + z_k$$

- Total transmit power of all the relays is $\sum_{k=1}^{K} \|\boldsymbol{v}_k\|^2 + \mathbf{Q} \bullet \mathbf{I}$
- SINR of user k is

$$\frac{|\boldsymbol{h}_k^\dagger \boldsymbol{v}_k|^2}{\sum_{j \neq k} |\boldsymbol{h}_k^\dagger \boldsymbol{v}_j|^2 + \boldsymbol{h}_k^\dagger \mathbf{Q} \boldsymbol{h}_k + \sigma^2}, \ \forall \ k \in \mathcal{K}$$

- Compression rate of relay m under the multivariate compression strategy [1] is

$$\log_2 \frac{\sum_{k=1}^{K} |v_{k,m}|^2 + \mathbf{Q}^{(m,m)}}{\mathbf{Q}^{(m:M,m:M)} / \mathbf{Q}^{(m+1:M,m+1:M)}}, \ \forall \ m \in \mathcal{M}$$

- $\mathbf{Q}^{(m:M,m:M)} / \mathbf{Q}^{(m+1:M,m+1:M)}$ is the Schur complement

Problem Formulation

The joint beamforming and compression problem [2]:

$$\min_{\{\boldsymbol{v}_k\},\mathbf{Q}} \quad \overbrace{\sum_{k=1}^{K} \|\boldsymbol{v}_k\|^2 + \mathbf{Q} \bullet \mathbf{I}}^{\text{total power}}$$

$$\text{s.t.} \quad \overbrace{\frac{|\boldsymbol{h}_k^\dagger \boldsymbol{v}_k|^2}{\sum_{j\neq k} |\boldsymbol{h}_k^\dagger \boldsymbol{v}_j|^2 + \boldsymbol{h}_k^\dagger \mathbf{Q} \boldsymbol{h}_k + \sigma^2}}^{\text{SINR of user } k} \geq \overbrace{\gamma_k}^{\text{SINR target of user } k}, \ \forall\ k \in \mathcal{K},$$

$$\overbrace{\log_2 \frac{\sum_{k=1}^{K} |v_{k,m}|^2 + \mathbf{Q}^{(m,m)}}{\mathbf{Q}^{(m:M,m:M)} / \mathbf{Q}^{(m+1:M,m+1:M)}}}^{\text{fronthaul rate of relay } m} \leq \overbrace{C_m}^{\text{fronthaul capacity of relay } m}, \ \forall\ m \in \mathcal{M},$$

$$\mathbf{Q} \succeq \mathbf{0}. \tag{1}$$

Problem Formulation

Equivalent formulation of (1) [2, Propostion 4]:

$$
\begin{aligned}
\min_{\{\boldsymbol{v}_k\},\mathbf{Q}} \quad & \sum_{k=1}^{K} \|\boldsymbol{v}_k\|^2 + \mathbf{Q} \bullet \mathbf{I} \\
\text{s.t.} \quad & \boldsymbol{v}_k^\dagger \mathbf{H}_k \boldsymbol{v}_k - \gamma_k \left(\sum_{j \neq k} \boldsymbol{v}_j^\dagger \mathbf{H}_k \boldsymbol{v}_j + \mathbf{Q} \bullet \mathbf{H}_k + \sigma^2 \right) \geq 0, \ \forall\, k \in \mathcal{K}, \\
& \eta_m \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}^{(m:M,m:M)} \end{bmatrix} - \mathbf{E}_m^\dagger \left(\sum_{k=1}^{K} \boldsymbol{v}_k \boldsymbol{v}_k^\dagger + \mathbf{Q} \right) \mathbf{E}_m \succeq \mathbf{0}, \\
& \qquad \forall\, m \in \mathcal{M}, \\
& \mathbf{Q} \succeq \mathbf{0},
\end{aligned}
\tag{P}
$$

where

- $\mathbf{H}_k = \boldsymbol{h}_k \boldsymbol{h}_k^\dagger$, $\eta_m = 2^{C_m}$.

Agenda

Design an efficient algorithm for solving (P)

1. Show zero-duality gap
   1. Derive the SDR of (P)
   2. Derive the dual problem of (P)
   3. Show that SDR is tight
2. Solve the KKT optimality conditions of the SDR based on its special structure

SDR of (P)

Semidefinite relaxation (SDR) of (P):

$$
\begin{aligned}
\min_{\{\mathbf{V}_k\},\mathbf{Q}} \quad & \sum_{k=1}^{K} \mathbf{V}_k \bullet \mathbf{I} + \mathbf{Q} \bullet \mathbf{I} \\
\text{s.t.} \quad & a_k(\{\mathbf{V}_k\},\mathbf{Q}) \geq 0, \quad \forall\, k \in \mathcal{K}, \\
& \mathbf{B}_m(\{\mathbf{V}_k\},\mathbf{Q}) \succeq \mathbf{0}, \quad \forall\, m \in \mathcal{M}, \\
& \mathbf{V}_k \succeq \mathbf{0}, \ \forall\, k \in \mathcal{K}, \\
& \mathbf{Q} \succeq \mathbf{0},
\end{aligned} \tag{2}
$$

where

$$
a_k(\{\mathbf{V}_k\},\mathbf{Q}) = \mathbf{V}_k \bullet \mathbf{H}_k - \gamma_k \left(\sum_{j \neq k} \mathbf{V}_j \bullet \mathbf{H}_k + \mathbf{Q} \bullet \mathbf{H}_k + \sigma^2 \right),
$$

$$
\mathbf{B}_m(\{\mathbf{V}_k\},\mathbf{Q}) = \eta_m \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}^{(m:M,m:M)} \end{bmatrix} - \mathbf{E}_m^{\dagger} \left(\sum_{k=1}^{K} \mathbf{V}_k + \mathbf{Q} \right) \mathbf{E}_m.
$$

Lagrangian Dual of (2)

The Lagrangian dual of problem (2):

$$
\begin{aligned}
\max_{\{\beta_k\},\{\mathbf{\Lambda}_m\}} \quad & \sum_{k=1}^{K} (\gamma_k \sigma^2)\beta_k \\
\text{s.t.} \quad & \mathbf{C}_k(\{\beta_k\},\{\mathbf{\Lambda}_m\}) - \beta_k \mathbf{H}_k \succeq \mathbf{0}, \quad \forall\, k \in \mathcal{K}, \\
& \mathbf{D}(\{\beta_k\},\{\mathbf{\Lambda}_m\}) \succeq \mathbf{0}, \\
& \beta_k \geq 0, \ \forall\, k \in \mathcal{K}, \\
& \mathbf{\Lambda}_m \succeq \mathbf{0}, \ \forall\, m \in \mathcal{M},
\end{aligned}
\tag{3}
$$

$$
\mathbf{C}_k(\{\beta_k\},\{\mathbf{\Lambda}_m\}) = \mathbf{I} + \sum_{m=1}^{M} \mathbf{E}_m^{\dagger} \mathbf{\Lambda}_m \mathbf{E}_m + \sum_{j \neq k} \beta_j \gamma_j \mathbf{H}_j,
$$

$$
\mathbf{D}(\{\beta_k\},\{\mathbf{\Lambda}_m\}) = \mathbf{I} - \sum_{m=1}^{M} \eta_m \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_m^{(m:M,m:M)} \end{bmatrix} + \sum_{k=1}^{K} \beta_k \gamma_k \mathbf{H}_k + \sum_{m=1}^{M} \mathbf{E}_m^{\dagger} \mathbf{\Lambda}_m \mathbf{E}_m.
$$

Tightness of SDR

$$\begin{aligned} \min_{V_k, Q} \quad & P^{\mathrm{dl}} \\ \text{s.t.} \quad & \beta_k : a_k \geq 0, \quad k \in \mathcal{K}, \\ & \mathbf{\Lambda}_m : \mathbf{B}_m \succeq \mathbf{0}, \quad m \in \mathcal{M}, \\ & \mathbf{V}_k, \ \mathbf{Q} \succeq \mathbf{0}, \quad k \in \mathcal{K}. \end{aligned} \tag{2}$$

$$\begin{aligned} \max_{\beta_k, \Lambda_m} \quad & P^{\mathrm{ul}} \\ \text{s.t.} \quad & \mathbf{V}_k : \mathbf{C}_k - \beta_k \mathbf{H}_k \succeq \mathbf{0}, \quad \forall k \in \mathcal{K}, \\ & \mathbf{Q} : \mathbf{D} \succeq \mathbf{0}, \\ & \beta_k \geq 0, \quad \forall k \in \mathcal{K}, \\ & \mathbf{\Lambda}_m \succeq \mathbf{0}, \quad \forall m \in \mathcal{M}. \end{aligned} \tag{3}$$

Theorem

Suppose that problem (2) is feasible. Then it always has a rank-one solution.

Agenda

Design an efficient algorithm for solving (P)

1. Show zero-duality gap
2. Solve the KKT optimality conditions of (2) based on its special structure
   1. Write out the equivalent KKT conditions
   2. Separate the equations into two sets and solve the equations involving the dual variables first and then the equations involving the primal variables
   3. Show that each set of equations can be solved elegantly via fixed-point iteration

KKT Conditions of SDR and Dual Problem

Equivalent KKT conditions:

$$
\begin{cases}
\mathbf{D}(\{\beta_k\}, \{\mathbf{\Lambda}_m\}) = \mathbf{0}, & (4) \\
\left.\begin{array}{l} \mathrm{rank}(\mathbf{\Lambda}_m) = 1,\ \mathbf{\Lambda}_m \succeq \mathbf{0},\ \forall\ m \in \mathcal{M}, \\ \mathbf{\Lambda}_m^{(1:m-1,1:m)} = \mathbf{0},\ \mathbf{\Lambda}_m^{(m:M,1:m-1)} = \mathbf{0},\ \forall\ m \in \mathcal{M}, \end{array}\right\} & (5) \\
\left.\begin{array}{l} \mathrm{rank}(\mathbf{C}_k(\{\beta_k\}, \{\mathbf{\Lambda}_m\}) - \beta_k \mathbf{H}_k) = M - 1,\ \forall\ m \in \mathcal{M}, \\ \mathbf{C}_k(\{\beta_k\}, \{\mathbf{\Lambda}_m\}) - \beta_k \mathbf{H}_k \succeq \mathbf{0},\ \forall\ m \in \mathcal{M}, \end{array}\right\} & (6) \\
\beta_k \geq 0,\ \forall\ k \in \mathcal{K}, & (7) \\
\mathbf{V}_k \bullet (\mathbf{C}_k(\{\beta_k\}, \{\mathbf{\Lambda}_m\}) - \beta_k \mathbf{H}_k) = 0,\ \forall\ k \in \mathcal{K}, & (8) \\
\mathbf{V}_k \succeq \mathbf{0},\ \mathrm{rank}(\mathbf{V}_k) = 1,\ \forall\ k \in \mathcal{K}, & (9) \\
a_k(\{\mathbf{V}_k\}, \mathbf{Q}) = 0,\ \forall\ k \in \mathcal{K}, & (10) \\
\mathbf{B}_m(\{\mathbf{V}_k\}, \mathbf{Q}) \succeq \mathbf{0},\ \forall\ m \in \mathcal{M}, & (11) \\
\mathbf{\Lambda}_m \bullet \mathbf{B}_m(\{\mathbf{V}_k\}, \mathbf{Q}) = 0,\ \forall\ m \in \mathcal{M}, & (12) \\
\mathbf{Q} \succeq \mathbf{0}. & (13)
\end{cases}
$$

Solving Eqs. (4)–(7): Eqs. (4) and (5)

- Given $\{\beta_k\}$
- Equivalent form of Eq. (4):

$$\sum_{m=1}^{M} \eta_m \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_m^{(m:M,m:M)} \end{bmatrix} - \sum_{m=1}^{M} \mathbf{E}_m^{\dagger} \mathbf{\Lambda}_m \mathbf{E}_m = \mathbf{I} + \sum_{k=1}^{K} \beta_k \gamma_k \mathbf{H}_k \triangleq \mathbf{\Gamma}$$

- only $\mathbf{\Lambda}_1$ affects the first row and column of matrix $\mathbf{\Gamma}$ $\Rightarrow$ the entries in the first row of $\mathbf{\Lambda}_1$ should be $\left[\frac{1}{\eta_1 - 1}\mathbf{\Gamma}^{(1,1)}, \frac{1}{\eta_1}\mathbf{\Gamma}^{(1,2:M)}\right]$
- $\mathbf{\Lambda}_1$ is of rank one (Eq. (5)) $\Rightarrow$ further obtain all entries of $\mathbf{\Lambda}_1$
- Subtract all terms related to $\mathbf{\Lambda}_1$:

$$\sum_{m=2}^{M} \eta_m \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_m^{(m:M,m:M)} \end{bmatrix} - \sum_{m=2}^{M} \mathbf{E}_m^{\dagger} \mathbf{\Lambda}_m \mathbf{E}_m = \mathbf{\Gamma} - \eta_m \mathbf{\Lambda}_1 + \mathbf{E}_1 \mathbf{\Lambda}_1 \mathbf{E}_1 \triangleq \mathbf{\Gamma}'$$

- Repeat the above procedure to find all $\mathbf{\Lambda}_m$ (which is also unique)
- Denote the solution to Eqs. (4)–(5) as $\{\mathbf{\Lambda}_m(\{\beta_k\})\}$

Solving Eqs. (4)–(7): Eqs. (6) and (7)

- Given $\{\mathbf{\Lambda}_m\}$
- Define $\mathbf{C}_k \triangleq \mathbf{C}_k(\{\beta_k\}, \{\mathbf{\Lambda}_m\})$. Recall Eq. (6):

$$\begin{cases} \mathrm{rank}(\mathbf{C}_k - \beta_k \mathbf{H}_k) = M - 1, \ \forall\ m \in \mathcal{M}, \\ \mathbf{C}_k - \beta_k \mathbf{H}_k \succeq \mathbf{0}, \ \forall\ m \in \mathcal{M} \end{cases}$$

- Notice that $\mathbf{H}_k \succeq \mathbf{0}$ is of rank one $\Rightarrow$ closed-form solution for β_k :

$$\beta_k \left(\{\mathbf{\Lambda}_m\}, \{\beta_j\}_{j \neq k} \right) = \left(\boldsymbol{h}_k^{\dagger} \mathbf{C}_k^{-1} \mathbf{h}_k \right)^{-1} > 0$$

Solving Eqs. (4)–(7) by Fixed-point Iteration

- Known $\{\beta_k\} \Rightarrow \{\mathbf{\Lambda}_m(\{\beta_k\})\}$ (Eqs. (4) and (5) holds)
- Known $\{\mathbf{\Lambda}_m\} \Rightarrow \left\{\beta_k\left(\{\mathbf{\Lambda}_m\}, \{\beta_j\}_{j\neq k}\right)\right\}$ (Eqs. (6) and (7) holds)
- $(\{\beta_k\}, \{\mathbf{\Lambda}_m(\{\beta_k\})\})$ that satisfy

$$\beta_k = I_k(\{\beta_k\}) \triangleq \beta_k\left(\{\mathbf{\Lambda}_m(\{\beta_k\})\}, \{\beta_j\}_{j\neq k}\right), \quad \forall\, k \in \mathcal{K} \tag{14}$$

  $\Rightarrow$ all Eqs. (4)–(7) holds
- Define $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_K]^\mathsf{T}$ and $I(\boldsymbol{\beta}) = [I_1(\{\beta_k\}), \ldots, I_K(\{\beta_k\})]^\mathsf{T}$, then (14) becomes

$$\boldsymbol{\beta} = I(\boldsymbol{\beta}). \tag{15}$$

Lemma

The function $I(\cdot)$ defined in (15) is a standard interference function.

- The fixed-point iteration $\boldsymbol{\beta}^{(i+1)} = I(\boldsymbol{\beta}^{(i)})$ will converge to the unique solution of (15). (Lemma and [4, Theorem 2])

Solving Eqs. (8)–(13)

- Given $\{\beta_k\}$ and $\{\mathbf{\Lambda}_m\}$ that satisfy Eqs. (4)–(7), find $\{\mathbf{V}_k\}$ and $\mathbf{Q}$ that satisfy Eqs. (8)–(13).
- Eqs. (8) and (9)

$$\mathbf{V}_k \bullet (\mathbf{C}_k - \beta_k \mathbf{H}_k) = 0, \ \forall\ k \in \mathcal{K}; \quad \mathbf{V}_k \succeq \mathbf{0}, \ \mathrm{rank}(\mathbf{V}_k) = 1, \ \forall\ k \in \mathcal{K}$$

$$\Rightarrow \boldsymbol{v}_k = \frac{\mathbf{C}_k^{-1}\boldsymbol{h}_k}{\|\mathbf{C}_k^{-1}\boldsymbol{h}_k\|}$$

- $\mathbf{U}_k = \boldsymbol{v}_k \boldsymbol{v}_k^\dagger$ (known), $\mathbf{V}_k = p_k \mathbf{U}_k$ ($\{p_k\}$ are the unknowns)
- Given $\mathbf{Q}$, Eq. (10) $\Rightarrow$

$$p_k\left(\mathbf{Q}, \{p_j\}_{j\neq k}\right) = \frac{\gamma_k \left(\sum_{j\neq k} p_j \mathbf{U}_j \bullet \mathbf{H}_k + \mathbf{Q} \bullet \mathbf{H}_k + \sigma^2\right)}{\mathbf{U}_k \bullet \mathbf{H}_k}$$

- Given $\{p_k\}$, Eqs. (11)-(13) $\Rightarrow$ $\mathbf{Q}(\{p_k\})$
- Fixed-point iteration $\boldsymbol{p}^{(i+1)} = J(\boldsymbol{p}^{(i)})$, standard interference function $J(\cdot)$ $\Rightarrow$ solves Eqs. (8)–(13)

Proposed Algorithm

- The algorithm first finds $\{\beta_k\}$ and $\{\mathbf{\Lambda}_m\}$ that satisfy Eqs. (4)–(7);
- With found $\{\beta_k\}$ and $\{\mathbf{\Lambda}_m\}$ fixed, the algorithm then finds $\{\mathbf{V}_k\}$ and $\mathbf{Q}$ that satisfy Eqs. (8)–(13).
- Hence, $\{\mathbf{V}_k\}$, $\mathbf{Q}$, $\{\beta_k\}$, and $\{\mathbf{\Lambda}_m\}$ together satisfy Eqs. (4)–(13) and thus is a KKT point of problem (2).
- Since $\mathrm{rank}(\mathbf{V}_k) = 1$ for all k, we can recover the optimal solution for problem (P).

Algorithm 1 Proposed Algorithm for Solving Problem (P)

1: Find $\{\beta_k\}$ and $\{\mathbf{\Lambda}_m\}$ that satisfy Eqs. (4)–(7) by performing the fixed-point iteration in (15) on $\{\beta_k\}$ until the desired error bound is met.
2: Find $\{\mathbf{V}_k\}$ and $\mathbf{Q}$ that satisfy Eqs. (8)–(13) by performing an appropriate fixed-point iteration on $\{p_k\}$ until the desired error bound is met.
3: Find $\boldsymbol{v}_k$ such that $\mathbf{V}_k = \boldsymbol{v}_k \boldsymbol{v}_k^\dagger, \ \forall\ k \in \mathcal{K}$.
4: **Output:** $\{\boldsymbol{v}_k\}$ and $\mathbf{Q}$.

Global Optimality Guarantee and Remarks

Theorem

If the SDR in (2) *is feasible, then proposed Algorithm 1 returns the optimal solution of problem (P).*

Remarks:

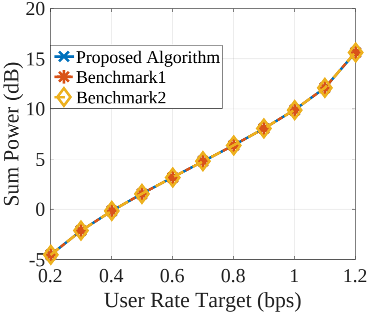
- Global optimality
- Computationally efficient: cheap evaluation in each step

Parameters Setting

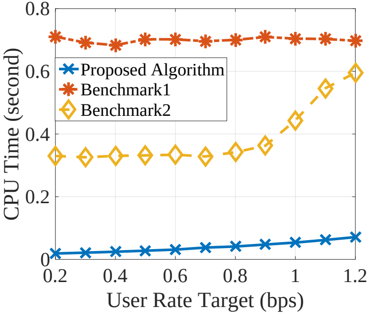
- Consider a network with
  - $M = 8$ relays and $K = 10$ users,
  - the wireless channels between these relays and users are generated based on the i.i.d. Rayleigh fading model following $\mathcal{CN}(0, 1)$,
  - and the fronthaul capacities between all relays and the CP are set to be 3 bits per symbol (bps).
- Moreover, the noise powers at the users are set to be $\sigma^2 = 1$.
- The rate targets for all the users are assumed to be identical.
- All simulation results are obtained by averaging over 200 Monte-Carlo runs.

Benchmarks

- Benchmark1: directly call CVX to solve the SDR in (2) $\Rightarrow$ verify the tightness
- Benchmark2: the proposed algorithm in [2] $\Rightarrow$ compare the efficiency
  - Fixed-point iteration $\Rightarrow$ dual uplink problem
  - Standard optimization solver (CVX) $\Rightarrow$ reduced primal downlink problem



(a) Average sum power versus the user rate target



(b) Average CPU time versus the user rate target.

- Fig. (a) verifies the tightness of the SDR and the global optimality of the solution returned by the proposed algorithm.
- Fig. (b) shows the high efficiency of our proposed algorithm.

Summary

- Propose an efficient and global algorithm for solving the downlink beamforming and compression problem
- Solve the KKT conditions by judiciously exploiting the problem structure
- Achieve the global optimality as the state-of-the-art algorithm proposed in [2] but with a significant less CPU time

Reference

S. H. Park, O. Simeone, O. Sahin, and S. Shamai, "Joint precoding and multivariate backhaul compression for the downlink of cloud radio access networks," *IEEE Trans. Signal Process.*, vol. 61, no. 22, pp. 5646-5658, Nov. 2013.

L. Liu, Y.-F. Liu, P. Patil, and W. Yu, "Uplink-Downlink duality between multiple-access and broadcast channels with compressing relays," *IEEE Trans. Inf. Theory*, vol. 67, no. 11, pp. 7304-7337, Nov. 2021.

X. Fan, Y.-F. Liu, and L. Liu, "Efficiently and Globally Solving Joint Beamforming and Compression Problem in the Cooperative Cellular Network via Lagrangian Duality," in Proc. IEEE Int. Conf. Acoustics, Speech, Signal Process. (ICASSP), Singapore, May, 2022.

R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1341-1347, Sept. 1995.