DISTRIBUTED BAYESIAN TRACKING ON THE SPECIAL EUCLIDEAN GROUP

USING LIE ALGEBRA PARAMETRIC APPROXIMATIONS

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1. Introduction

- The joint rotational and translational state of a rigid body can be parameterized as an element of the Special Euclidean Group \(\text{SE}(n)\).
- Modern engineering systems involve cooperation between multiple agents on a partially connected network to run a common task, e.g., estimate a hidden state.
- In previous works, we introduced diffusion particle filters (PF) to perform cooperative tracking of states that evolved on the Spherical and the Stiefel manifolds and the Special Orthogonal Group.
- Diffusion PFs include a data assimilation step where agents update their beliefs about the unknown states, assimilating local measurements and measurements from neighboring nodes.
- The local updated beliefs are then exchanged between nodes in a compressed form, using Gaussian parametric approximations on the Lie Algebra associated to \(\text{SE}(n)\).

2. Special Euclidean Group \(\text{SE}(n)\)

- The Special Euclidean Group \(\text{SE}(n)\) is a matrix Lie group. An element \(X\) of \(\text{SE}(n)\) is given as
  \[
  X = \begin{bmatrix}
  A & b \\
  0 & 1
  \end{bmatrix}
  \]
  where \(A\) is a \(n\times n\) matrix, \(b\) is a \(n\times 1\) vector, and \(0_{n\times 1}\) denotes a vector with null entries.
- The group \(\text{SE}(n)\) has dimension \(n^2 + n\), and for \(n = 1\), it corresponds to the set of all possible translations and rotations of a 3-dimensional rigid object.
- \(\text{SE}(n)\) is also a differentiable manifold. Thus, we can define a tangent space \(T_X\) at any point \(X \in \text{SE}(n)\).
- The Lie algebra \(\mathfrak{se}(n)\) is, by definition, the tangent space to the identity matrix \(I\), i.e., \(T_I\).
- A matrix \(X \in \text{SE}(n)\) can be mapped into a matrix \(Y \in \mathfrak{se}(n)\) using the log map \(\text{Log} : \text{SE}(n) \rightarrow \mathfrak{se}(n)\).
- The Lie algebra \(\mathfrak{se}(n)\) is a subalgebra of \(\mathfrak{gl}(n)\), and for \(n = 1\), it is isomorphic to \(\mathbb{R}^2\).

3. Problem Setup

- Let \(S_t \in \text{SE}(n)\) denote an unknown state at time \(t \geq 0\) that evolves according to the random walk
  \[
  S_t = S_{t-1} \exp (\Theta_t^i), \quad k > 0,
  \]
  with \(p(S_0) = 1\), where \(\{\Theta_t^i\}\) is a sequence of i.i.d. Gaussian random vectors in \(\mathbb{R}^n\) with zero mean and covariance matrix \(A_0\).
- The nodes record at each instant \(t\) the observations
  \[
  Y_t^i = H_t S_t + W_t^i, \quad k > 0,
  \]
  where \(r \in \{1, \ldots, R\}\) denotes the \(r\)-th node in the network, \(H_t^i \in \mathbb{R}^{m \times n}\) is a general function, \(W_t^i \in \mathbb{R}^{m \times 1}\) is a \(n\times 1\) identity matrix without its bottommost row, and \(\{W_t^i\}\) is a sequence of i.i.d. samples of a Matrix Gaussian p.d.f. \(N_{\mathbb{R}^{m \times 1}}(\mathbf{0}, \Omega_t^i)\).
- Given the observations \(\{Y_t^1, \ldots, Y_t^R\}\) with \(k \leq l < r \leq L\), our goal is to recursively estimate \(S_l\) in a distributed fashion.

4. RndEx Diffusion PF

- The RndEx Algorithm has two steps: Random Exchange and Data Assimilation.
- In the Random Exchange step, a node \(r\) exchanges with another randomly chosen node \(i\) its posterior p.d.f. \(p(S_l | Y_{1:l}, Y_{1:i-1}, Y_{i+1:L})\), in which \(S_{l-1}^{i-1}\) denotes all observations assimilated up to instant \(i\).
- Suppose that the posterior p.d.f. is approximated by the weighted particle set \(\{q_{l}^{i}, S_{l-1}^{i}\}\), \(q_1, \ldots, Q \gg 1\).
- Before the exchange, node \(i\) compresses the representation using a Gaussian parametric approximation as follows.
  1) Given \(\{q_{l}^{i}, S_{l-1}^{i}\}\) compute its centroid \(S_{l-1}^{i-1}\) as
  \[
  S_{l-1}^{i-1} = \sum_1 Q \frac{q}{q_{i}^{i}} \log \left( \frac{q_{l}^{i}}{q_{r}^{i}} \right) \Omega_{l}^{i}^{-1} \Omega_{l-1}^{i-1},
  \]
  where \(S_{l-1}^{i-1}\) denotes the \(l\)-th estimate of the weighted average, with \(\Omega_{l}^{i-1}\) chosen as a random element of the particle set.
  2) Then, we evaluate \(\hat{S}_{l-1}^{i-1} = \hat{Q} \log \left( \frac{q_{l}^{i}}{q_{r}^{i}} \right) \Omega_{l}^{i-1} \Omega_{l-1}^{i-1}\), where \(\hat{Q} = \sum_1 Q \frac{q}{q_{r}^{i}} \log \left( \frac{q_{l}^{i}}{q_{r}^{i}} \right) \Omega_{l}^{i-1} \Omega_{l-1}^{i-1}\).
- The weighted particle set \(\{q_{l}^{i}, S_{l-1}^{i-1}\}\) is then approximated by a Gaussian p.d.f. via moment matching, with sample moments \(\hat{S}_{l-1}^{i-1}\) and \(\hat{\Omega}_{l}^{i-1}\).