Sample Space-Time Covariance Estimation

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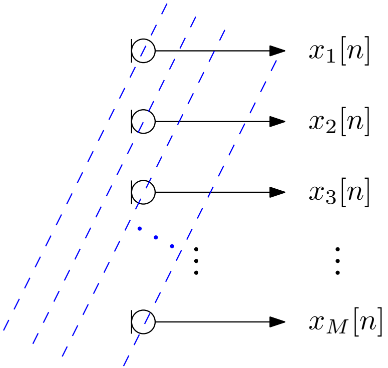
44th International Conference on Acoustics, Speech, and Signal Processing
16th May 2019, Brighton

Presentation Overview

1. Background Theory & Motivation
2. Source Model and Space-Time Covariance Matrix
3. Estimation
4. Variance of the Estimate and Results
5. Application of Research
6. Conclusion

Background

- In the field of broadband array processing, we can use polynomial matrices to extend narrowband problems to the broadband domain
- For the past decade, research has been focused on the polynomial eigenvalue decomposition (PEVD)
- Two well-known time-domain iterative algorithms, SBR2[1] and SMD[2], have been developed
- Many variations on these have been created
- Space-time covariance matrices comprise of auto- and cross-correlation sequences



[1]Second-Order Sequential Best Rotation Algorithm
[2]Sequential Matrix Diagonalisation

Motivation

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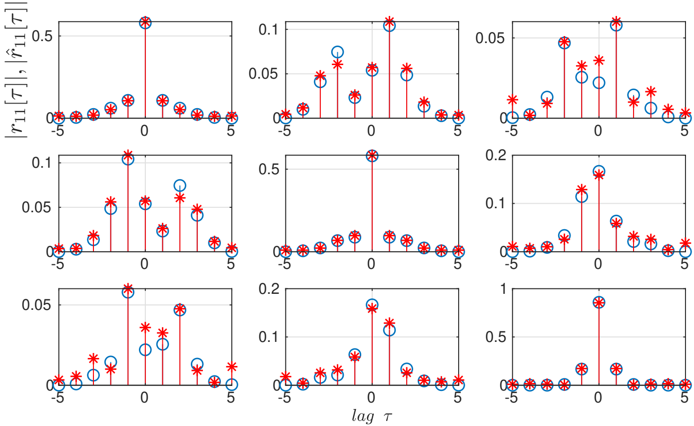
- The performance of these methods is of importance to a number of applications
- We have a gap in the connection between theory and practice

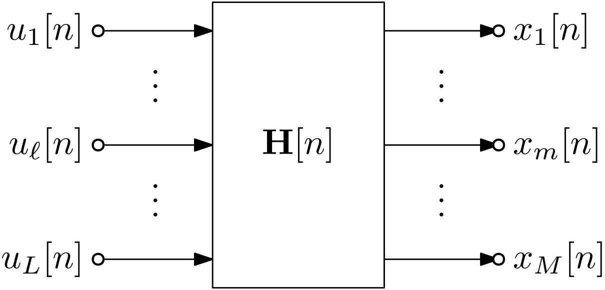


Source Model

- We express the data vector $\mathbf{x}[n]$ through a source model
- As a result of the source model used, the ground truth S-T covariance matrix is expressed as

$$\mathbf{R}[\tau] = \mathbf{H}[\tau] * \mathbf{H}^{\mathrm{H}}[-\tau] \tag{1}$$

- This matrix satisfies the symmetry property $\mathbf{R}[\tau] = \mathbf{R}^{\mathrm{H}}[-\tau]$



Sample Space-Time Covariance Matrix

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- $\mathbf{R}[\tau]$ is not available in practice
- The cross-correlation must be unbiased in order for a rank-M matrix to exist
- We can calculate the error matrix, $\mathbf{E}[\tau] = \hat{\mathbf{R}}[\tau] - \mathbf{R}[\tau]$, as a measure of the perturbation in our ground-truth
- The effect of the perturbation of eigenvalues and -vectors due to this estimation has been investigated

$$\hat{r}_{m\mu}[\tau] = \begin{cases} \frac{1}{N-|\tau|} \sum_{n=0}^{N-|\tau|-1} x_m[n+\tau] x_\mu^*[n] \,, & \tau \geq 0 \\ \frac{1}{N-|\tau|} \sum_{n=0}^{N-|\tau|-1} x_m[n] x_\mu^*[n-\tau] \,, & \tau < 0 \end{cases} \quad (2)$$

Variance of the Estimator

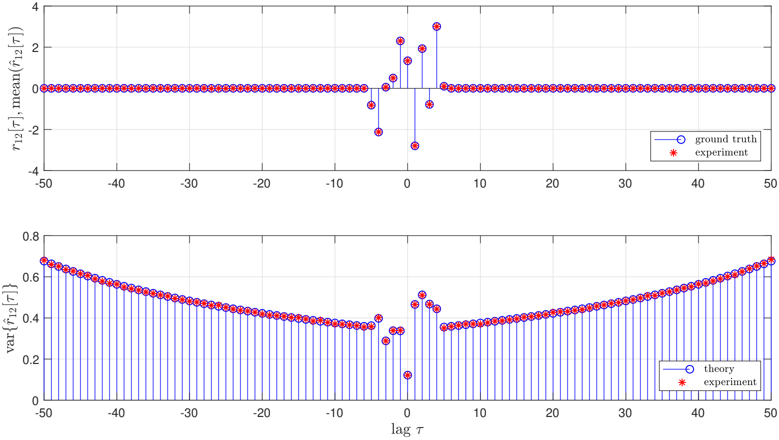
- The estimate is dependent on the sensor data
  - Still the same underlying ground truth
- The cross correlation can be measured across ensembles where the distribution was previously unknown
  - Spatial-only lag ($\tau = 0$) is known to be Wishart distributed
- The variance of our estimator is derived to be

$$\text{var}\{\hat{r}_{m\mu}[\tau]\} = \frac{1}{(N-|\tau|)^2} \sum_{t=-N+|\tau|+1}^{N-|\tau|-1} (N - |\tau| - |t|) \cdot \cdot \left(r_{mm}[t] r^*_{\mu\mu}[t] + \bar{r}_{m\mu}[\tau + t] \bar{r}^*_{m\mu}[\tau - t] \right) \tag{3}$$

where $\bar{r}_{m\mu}[\tau] = \mathcal{E}\{x_m[n] x_\mu[n - \tau]\}$

Variance of Estimate: Real-Valued data

- For real-valued $\mathbf{x}_m$ and $\mathbf{x}_\mu$ with $L = 1$, $N = 100$ and over an ensemble of size 10^4



Variance of Estimate: Complex-Valued data

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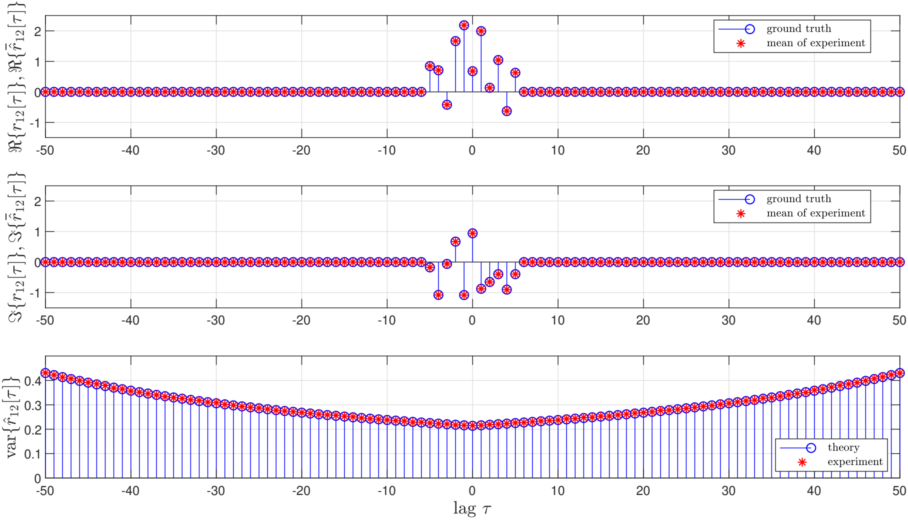
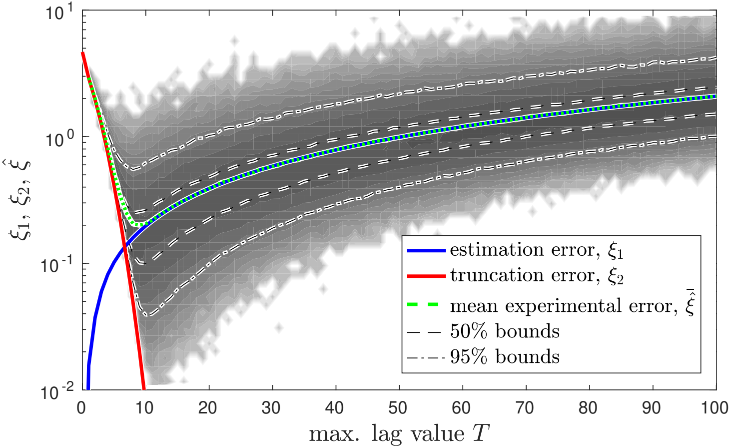
- For complex-valued $\mathbf{x}_m$ and $\mathbf{x}_\mu$



Optimum Support Length

- We define the mean squared error, ξ, which comprises of the estimation error and truncation error i.e. $\xi = \xi_1 + \xi_2$
- $T_{opt} = \arg\min_T \xi$



Conclusion

We have discussed:

- Background of Polynomial/Parahermitian Matrices
- Ground-truth model and construction of the true space-time covariance matrix
- Definition and statistics of an estimated space-time covariance matrix
- Demonstration of simulation results for mean and variance (experimental vs theoretical)
- Application of research – Optimum Support Length