Switched Hawkes Processes
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Hawkes Processes

A class of auto-regressive point processes used to model data in which events tend to cluster and influence the likelihood of future events.

Applications

- Earthquakes
- Finance
- Social media

Motivation

Majority of the works on Hawkes Processes assume that the parameters determining the intensity function remain constant.

But, in most of the practical scenarios, the underlying dynamics which influence the nature of the process can change over time.

An example - traffic flow analysis

Two representative states of traffic flow at an intersection:

State 1

State 2

- Squares numbered 1 - 16 represent sensors: they record an event whenever a vehicle passes over them.
- State 1 - events recorded by sensors 13, 3, . . . and correlations between events of sensors 13 and 4, 3 and 16, . . . but that does not hold good for State 2
- The nature of events observed depends on the underlying dynamics - in this case being the traffic lights
- A static model would fail to explain such observations

Switched Model

We propose a switched model - the overall process switches among a known finite number of states and every state is characterized by an individual process

Conditional Intensity Function for an N-dimensional process of order M

\[ \lambda_n(t|H_t) = \mu_n + \sum_{j=1}^{N} \sum_{k 
eq n} \sum_{r_k \in H_t \cap (t-k,t]} \gamma(t-k) \]

where \( H_t \cap (t-k,t] = \{ \tau_k : \tau_k < t \} \)

\( \mu_n \) - base intensity
\( \gamma(t) \) - excitation co-efficients

Here, \( \gamma(t) \) characterize the inter-process influence.

Traffic Data – QQ plots

- We consider traffic data at an intersection and investigate which of the two models better explains the data.
- We estimate a 16-dimensional switched Hawkes model of order 6 and a 16-dimensional static Hawkes model.

Maximum Likelihood Parameter Estimation

Estimation of \( \mu, A \)

Suppose that we observe the sequence of events \( \{(\tau_1, \theta_1, s_1), \ldots, (\tau_K, \theta_K, s_K)\} \) on \([0, T]\). The likelihood of these events is given by

\[ L(\mu, A|H_T) = \prod_{k=1}^{K} \lambda_{s_k}(\tau_k) \exp \left( - \int_{0}^{T} \lambda(t)dt \right) \]

where \( \lambda(t) = \sum_{n=1}^{M} \lambda_n(t) \)

Separable form

\[ L(\mu, A|H_T) = \sum_{k=1}^{N} \lambda_{s_k}(\mu, A|H_T) \]

where

\[ \lambda_{s_k}(\mu, A|H_T) = \int_{0}^{T} \lambda_{s_k}(t)dt - \sum_{\tau_k \in H_T \cap (t-k,t]} \log \lambda_{s_k}(\tau_k) \]