Multicarrier radar-communications waveform design for RF-convergence and coexistence

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RF Convergence

- Using the same platform for multiple uses
  - Radar and communications tasks

- Different approaches:
  - Tasks are performed sequentially (time division)
  - Tasks are performed simultaneously by exploiting different DoFs:
    - Frequency
    - Antenna elements
    - Radiation patterns (comms or radar in sidelobes)
  - Integrated waveforms (one waveform for both tasks, e.g. embedding data symbols into the radar waveform → very common)
Proposed approach

- Use multicarrier waveforms for which interleaved subcarriers or subsets of subcarriers can be assigned to different tasks

- Objectives:
  - Devise a strategy to assign subcarriers to either task
  - Optimize the power use for each task
Performance metric

- Mutual Information (MI) is chosen as the performance metric for our objectives
- Is MI a suitable metric?
  - For comms it is directly related to the capacity
    \[ C = \max I(X; Y), \quad X \text{ – Channel input, } Y \text{ – Channel output} \]
  - For radar MI maximization has been connected to minimum mean square error (MMSE) and was shown to also provide waveforms with good detection properties
    \[ \max I(Y; H), \quad Y \text{ – Received signal, } H \text{ – Target impulse response} \]
System model

- A dual-use radar-communications OFDM waveform is considered
- Waveform is reflected off the target and received by the communications user
- Waveform can be modeled as follows:

\[ x = F^H (W r + (I - W)c) , \]

- \( F^H \) – IDFT matrix
- \( W \) – \( N \times N \) diagonal subcarrier selection matrix (0s and 1s)
- \( r \) – Radar transmitted symbols
- \( c \) – Communications transmitted symbols
Compound objective function

- A **compound MI** based objective function is formulated for subcarrier assignment and optimum power allocation

\[
I(y_r; h_r) + I(y_c; x),
\]

- **radar MI**
- **comms MI**

- \(y_r\) — Signal at the radar receiver
- \(h_r\) — Target impulse response
- \(y_c\) — Signal at the communications receiver
- \(x\) — Transmitted dual-use waveform

\[
\frac{1}{2} \left[ \sum_{k=0}^{N-1} \log \left( 1 + \frac{w^2[k]|r[k]|^2 \sigma_{h_r}^2[k]}{u^2[k]|c[k]|^2 \sigma_{h_r}^2[k] + \sigma_n^2} \right) + \sum_{k=0}^{N-1} \log \left( 1 + \frac{u^2[k]|c[k]|^2 \sigma_{h_c}^2[k]}{w^2[k]|r[k]|^2 \sigma_{h_c}^2[k] + \sigma_m^2} \right) \right]
\]
Compund objective function

For any $k$th subcarrier only $w$ or $u$ can be non-zero, thus the objective function can be simplified as follows:

$$\frac{1}{2} \left[ \sum_{k=0}^{N-1} \log \left( 1 + \frac{w[k]|r[k]|^2 \sigma_{hr}^2[k]}{\sigma_n^2} \right) + \sum_{k=0}^{N-1} \log \left( 1 + \frac{u[k]|c[k]|^2 \sigma_{hc}^2[k]}{\sigma_m^2} \right) \right],$$

- $w[k]$ – Radar weight on $k$th subcarrier - $\{0, 1\}$
- $u[k]$ – Comms weight on $k$th subcarrier - $\{0, 1\}$
- $\sigma_{hr}^2[k]$ – Radar channel gain on $k$th subcarrier
- $\sigma_{hc}^2[k]$ – Comms channel gain on $k$th subcarrier
- $\sigma_n^2$ – Noise power @ radar receiver
- $\sigma_m^2$ – Noise power @ comms receiver
Proposed design algorithms

- Two design algorithms are proposed:
  - “Radar selfish design”
  - “Cooperative design”

- A brief description of the two algorithms:

<table>
<thead>
<tr>
<th>”Radar selfish design”</th>
<th>”Cooperative design”</th>
</tr>
</thead>
<tbody>
<tr>
<td>▶ Radar receives all subcarriers</td>
<td>▶ Allocate subcarriers to either subsystem based on MI objective</td>
</tr>
<tr>
<td>▶ Optimize radar power based on MI objective</td>
<td>▶ Optimize power for each subsystem</td>
</tr>
<tr>
<td>▶ Allocate unused subcarriers to comms. subsystem</td>
<td></td>
</tr>
<tr>
<td>▶ Minimize the number of radar subcarriers</td>
<td></td>
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<tr>
<td>based on some allowed loss of radar MI</td>
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<tr>
<td>▶ Obtain final allocation of subcarriers</td>
<td></td>
</tr>
<tr>
<td>▶ Optimize power for each subsystem</td>
<td></td>
</tr>
</tbody>
</table>
Radar selfish design

- **Radar power allocation optimization** is formulated as:
  
  \[
  \text{maximize} \quad \sum_{k=0}^{N-1} \log \left(1 + \frac{w[k]|r[k]|^2 \sigma_n^2}{\sigma_{h,r}^2[k]} \right) \quad \leftarrow \text{Radar MI}
  \]

  subject to \quad \sum_{k=0}^{N-1} w[k]|r[k]|^2 \leq P_T \quad \leftarrow \text{Total radar power budget}

- **Comms power allocation optimization** is formulated as:
  
  \[
  \text{maximize} \quad \sum_{k=0}^{N-1} \log \left(1 + \frac{u[k]|c[k]|^2 \sigma_n^2}{\sigma_{h,c}^2[k]} \right) \quad \leftarrow \text{Comms MI}
  \]

  subject to \quad \sum_{k=0}^{N-1} u[k]|c[k]|^2 \leq P_T \quad \leftarrow \text{Total comms power budget}
Radar selfish design

- Both optimization problems can be solved exactly and their solutions are *water filling* solutions.
- Subcarriers with higher channel gain and low noise and interference power receive more power.
- Example of power allocation for both subsystems before minimizing the number of radar subcarriers.
Radar selfish design

• Minimizing the number of radar subcarriers is done so that comms can receive more subcarriers
• This problem is non-convex ($\ell_0$-norm minimization)
• The best convex approximation is used ($\ell_1$-norm minimization)

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=0}^{N-1} w[k] \\
\text{subject to} & \quad \sum_{k=0}^{N-1} \log \left( 1 + \frac{w[k] |r[k]|^2 \sigma_{n_r}^2[k]}{\sigma_r^2} \right) \geq t \\
& \quad 0 \leq w[k] \leq 1, \forall k = 0 \ldots N - 1
\end{align*}
\]

Radar MI

From previous step

$\text{Radar MI}$

t is chosen 5 – 25% smaller than the initial maximum MI

• A rounding to 0 or 1 is used to obtain the actual $w$
• Iterate until no change in $w$
Radar selfish design

- An example of subcarrier allocation and power optimization for both subsystems at the initial and final steps
Radar selfish design

- This design involves a trade-off between MI loss for radar and MI gain for the communications subsystems.
- The average MI change from first to last step of the algorithm for different number of subcarriers (500 channel realizations) shows that it pays off to allow a small decrease in radar maximized MI for a larger comms maximized MI → higher capacity.
Cooperative design

• Subcarriers are assigned to the radar or the comms subsystem based on maximizing the compound objective.

• The objective can be further simplified to:

\[
\frac{1}{2} \left[ \sum_{k=0}^{N-1} \log \left( 1 + \frac{w[k] \sigma_{hr}^2[k]}{\sigma_n^2} + \frac{u[k] \sigma_{hc}^2[k]}{\sigma_m^2} \right) \right]
\]

• It turns out the optimum \( w \) and \( u \) are given by:

\[
\begin{cases}
\text{If } \frac{\sigma_{hr}^2[k]}{\sigma_n^2} > \frac{\sigma_{hc}^2[k]}{\sigma_m^2} & \text{then } w[k] = 0, u[k] = 1 \\
\text{If } \frac{\sigma_{hr}^2[k]}{\sigma_n^2} \leq \frac{\sigma_{hc}^2[k]}{\sigma_m^2} & \text{then } w[k] = 1, u[k] = 0
\end{cases}
\]

• Subcarriers go to the subsystems that experience larger “channel to noise” ratio.
Cooperative design

- Example of final power allocation for both subsystems, which is different than for the first design algorithm
Radar selfish vs cooperative design

- Comparing the maximized MI achieved using both strategies
  - Cooperative design is favorable to the comms subsystem as expected
  - Radar selfish design is favorable to the radar subsystem as long as there is not too much MI loss allowed
Thank you!