Optimal Power Flow Using Graph Neural Networks

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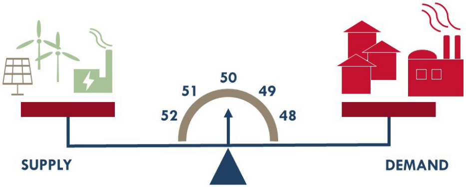

- Requires a constant balance between supply and demand $\Rightarrow$ supply = demand

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- Use **graph neural networks** to learn the optimal power allocation in a network.
  - $\Rightarrow$ Local computations, distributed implementations, scalability

Optimal Power Flow

Electrical Grids as Graphs

Graph Neural Networks

Imitating Optimal Power Flow

Conclusions

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$$\mathbf{X} = \begin{bmatrix} \mathbf{v} & \boldsymbol{\delta} & \mathbf{p} & \mathbf{q} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1^T & \dots & \mathbf{x}_n^T & \dots & \mathbf{x}_N^T \end{bmatrix}^T$$

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- The physical characteristics of the grid are described by the power flow equations

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$\Rightarrow$ Relate local net power generation with the global state

$\Rightarrow$ Depends on the topology $\mathbf{W}$ of the grid $\mathbf{W}$ is the set of electrical components in the grid

- Thus optimal power flow is a minimization problem

$$\underset{\{p_n\}}{\text{minimize}} \sum_{n=1}^{N} c_n(p_n, q_n) \tag{1}$$

(5)

- Where,

  $\Rightarrow$ $c_n(p_n, q_n)$ is the cost to generate power at the nth node

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$$\text{subject to } \mathbf{p} = \mathcal{P}(\mathbf{v}, \delta; \mathbf{W}), \tag{2}$$

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- Where,
  - $\Rightarrow$ $c_n(p_n, q_n)$ is the cost to generate power at the nth node
  - $\Rightarrow$ (2) and (3) are the powerflow equations $\Rightarrow$ sinusoidal constraints
  - $\Rightarrow$ $\mathbf{X}^{min}$ and $\mathbf{X}^{max}$ collect the minimum and maximum values each state entry can take
  - $\Rightarrow$ $\mathbf{X} \preceq \mathbf{Y} \iff [\mathbf{X}]_{ij} \leq [\mathbf{Y}]_{ij}$

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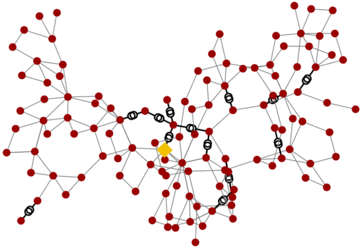
ACOPF

- Provides **exact** solution to OPF
- Solved using interior point methods (IPOPT)
- Very slow for large networks

  $\Rightarrow$ Impractical for real-time optimization

- Electrical grid $\Rightarrow$ weighted graph



IEEE118 Power System Network

- Electrical grid $\Rightarrow$ weighted graph
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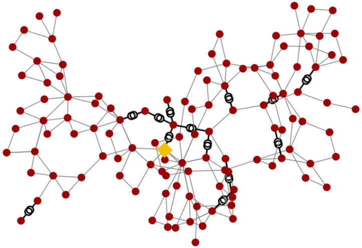

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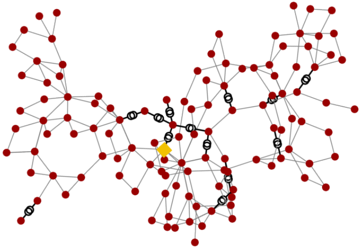

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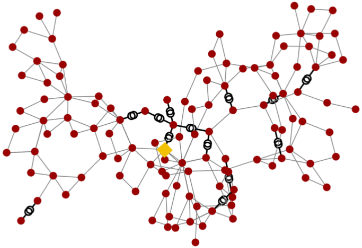
- State matrix $\mathbf{X} \in \mathbb{R}^{N \times 4}$ = graph signal with 4 features
  - $\Rightarrow$ Each row is the state of the corresponding node



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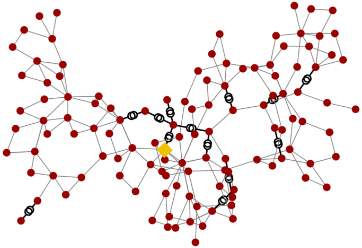

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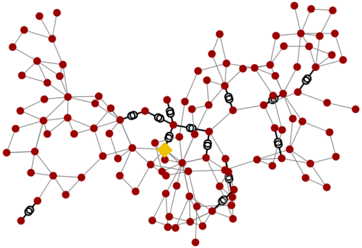
$$\mathbf{A}_{ij} = exp(-k|z_{ij}|^2)$$

- Where z_{ij} is the line impedance



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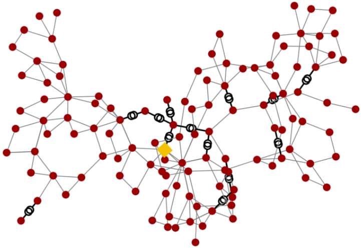

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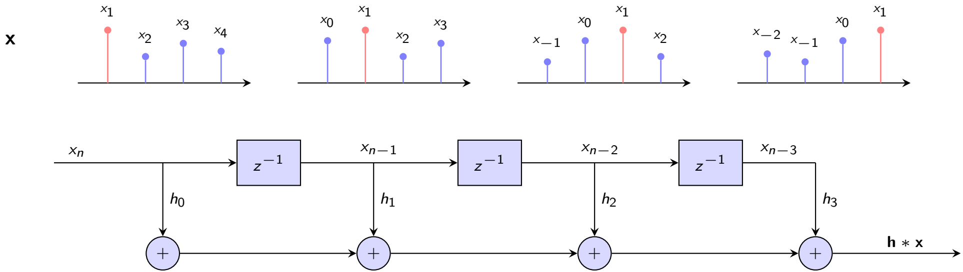
$$\mathbf{x} * \mathbf{h} = \sum_{k=0}^{K-1} h_k x_{n-k}$$



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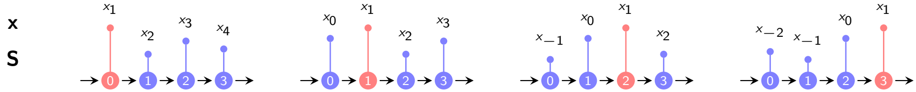

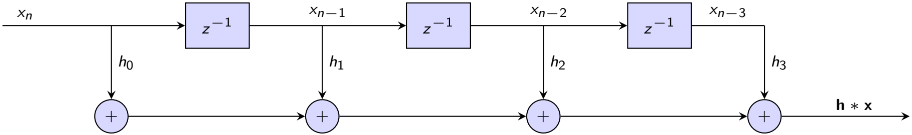

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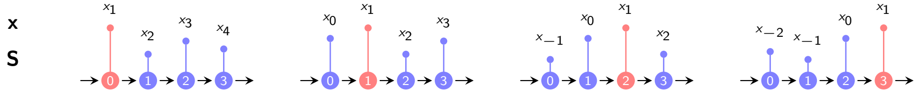

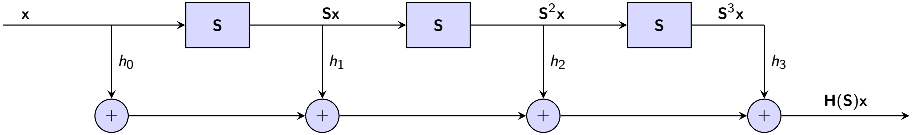

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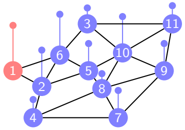

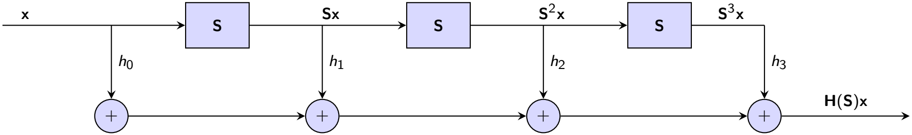

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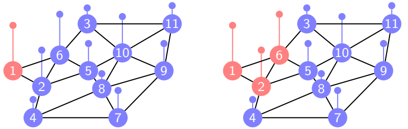

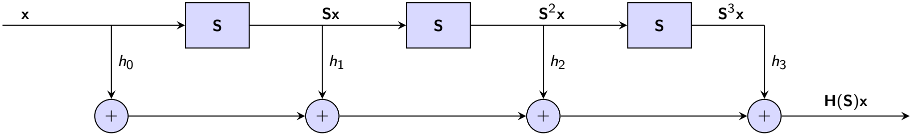

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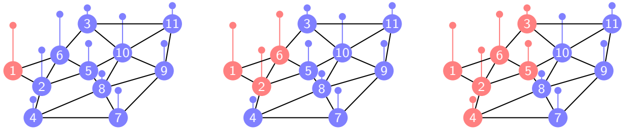

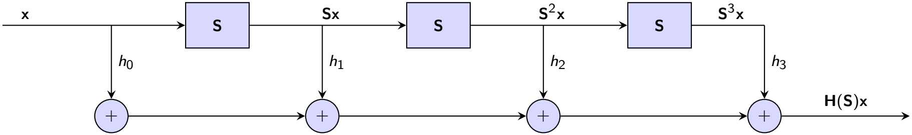

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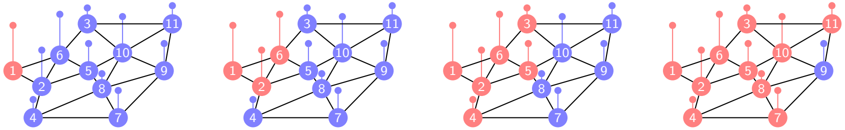

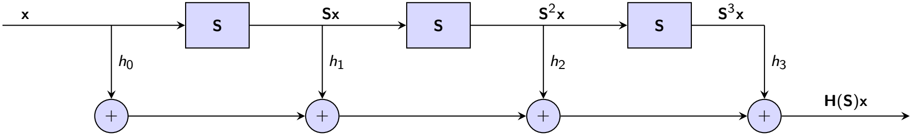

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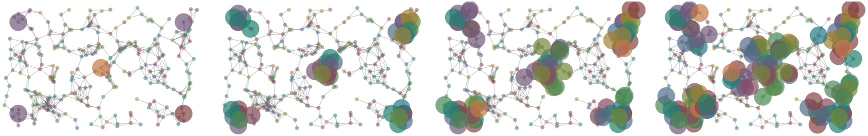
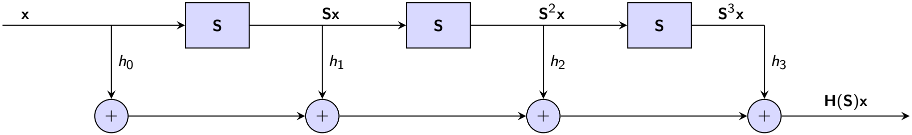
- Notion of shift $\mathbf{S}$ $\Rightarrow$ Matrix description of graph (adjacency, Laplacian)
- Linear combination of neighboring signal $\Rightarrow$ Local operation

$(\mathbf{x}; \mathbf{S})$





Gama, Marques, Leus, Ribeiro, "Convolutional Graph Neural Networks", Asilomar, 2019.

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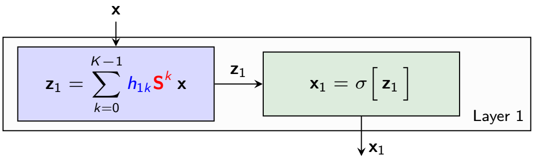

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Graph Neural Networks

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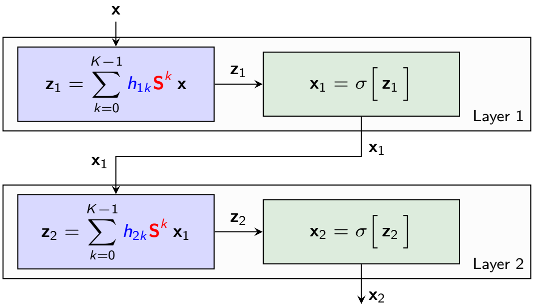

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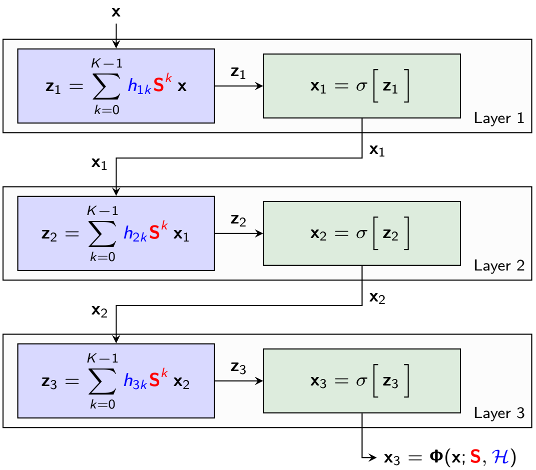

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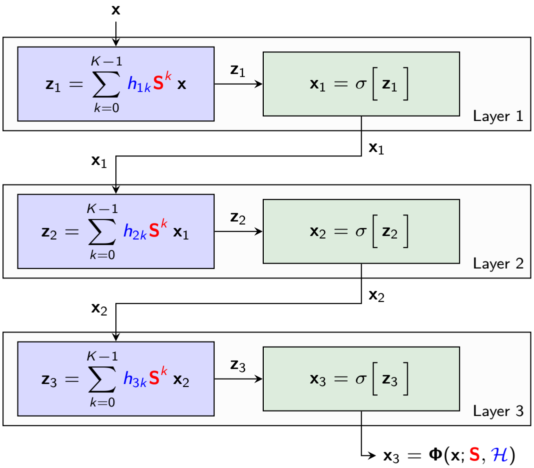

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Imitating Optimal Power Flow

- We compare GNNs against Multi-Layer Perceptrons

Global vs Local MLP

Note the difference between the Global and Local MLP. In the latter architecture there are N independent neural networks, one for each node.

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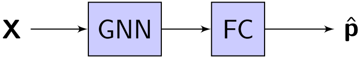

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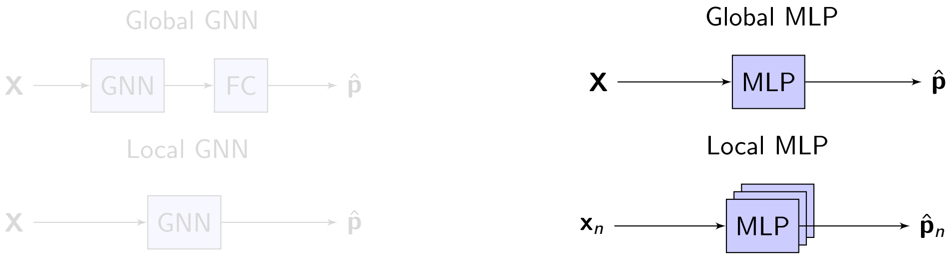

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  - $\Rightarrow$ The GNN takes $\approx 50\mu s$ to make predictions $\Rightarrow$ GNNs are 10^5 times faster

Conclusions

- Solving OPF is central to electrical grid operation
- OPF $\Rightarrow$ How to satisfy demand while **minimizing** operational costs?

  $\Rightarrow$ Non-linear constrains $\Rightarrow$ computationally expensive (NP hard)
- GNNs are well suited to applications on the electrical grid

  $\Rightarrow$ Scalable $\Rightarrow$ number of taps independent on network size

  $\Rightarrow$ Exploit the network structure of the data
- GNNs are up to 10^5 times faster than IPOPT

Thank You!