EXACT SPARSE SUPER-RESOLUTION VIA MODEL AGGREGATION

Paper #2679

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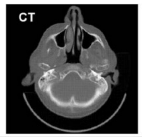
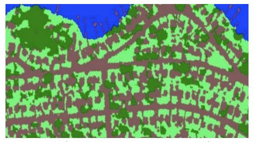
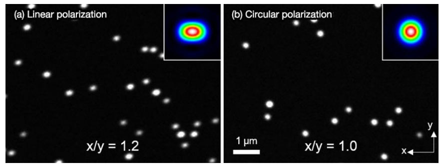
1.Application and Goal


MRI


Land segmentation


Super stars images

Goal: apply separation prior in algorithm to improve accuracy

2.Motivation: Separation is the Key

Theory

D.L. Donoho, 1990

$$\lim_{r\to\infty} r^{-1}\sup_t \#(S\cap[t,t+r)) < \frac{\Omega}{\pi}$$

Emmanuel J. Candès et al., 2012

$$\Delta(T)\geq 2.38/f_c = 2.38\lambda_c$$

Wenjing Liao et al., 2021

$$d(\omega_j,\omega_l) > \frac{1}{L}\sqrt{\frac{2}{\pi}}\left(\frac{2}{\pi}-\frac{1}{L}-\frac{8s}{\pi L^2}\right)^{-\frac{1}{2}}$$

Algorithm

$$\min_x \|x\|_1 \ \text{s.t.}\ Ax=y$$

$$\min_{x\in R^N}\{\|y-Ax\|_2^2+\lambda\|x\|_1\}$$

ESPRIT $\hat{\phi}=(\hat{S}_1^*\hat{S}_1)^{-1}\hat{S}_1^*\hat{S}_2$

spectral MUSIC $\dfrac{1}{a^*(\omega)\hat{G}\hat{G}^*a(\omega)}$

Separation Blind

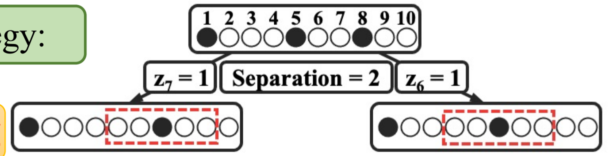
5. Estimation by Sampling

Closed-form: $\hat{\lambda}_l^{EW}=\dfrac{\exp(-M\hat{r}_l/\beta)\pi_l}{\sum_{k=1}^L \exp(-M\hat{r}_k/\beta)\pi_k}$

Computational Issue — **Too Large:** $L\sim O(2^R)$

Posterior Sampling: $\lambda_l \propto \exp(-M\hat{r}_l/\beta)\pi_l,\ \mathbf{p}_l\in\tilde{\mathcal{P}}$

Searching Strategy:



$\tilde{\mathcal{P}}$: separation prior

3.Main Idea: Estimation via Aggregating Sub-Models

A known family $\mathcal{F}$ of a **priori** hypotheses on the signal f: $\mathcal{F}=\{f_1,f_2,\dots,f_L\}$

Convex combine: $\boldsymbol{f}_\lambda(X)\triangleq\sum_{l=1}^L\lambda_l f_l(X),\ \forall X\in\mathcal{X}$

Weight : $\Lambda=\left\{\lambda=(\lambda_1,\dots,\lambda_L)^\top\in\mathbb{R}^L:\lambda_j\geq 0,\sum_{j=1}^L\lambda_j=1\right\}$

Sub model: $\mathbf{supp}=\mathbf{p}\in\mathcal{P}\triangleq\{0,1\}^N$

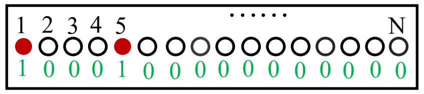
$\boldsymbol{f}_l(X)\triangleq \mathbf{A}_l\hat{\mathbf{z}}_l \rightarrow$

$\hat{\mathbf{z}}_l=\arg\min\|\mathbf{y}-\mathbf{A}_l\mathbf{z}\|_2$

$\mathcal{Z}_l\triangleq\{\mathbf{z}\odot\mathbf{p}_l,\mathbf{z}\in\mathbb{C}^N\}\subset\mathbb{C}^N$

$\mathbf{A}_l=[A^1_{\{:\}},0_{\{:\}},0_{\{:\}},A^5_{\{:\}},0_{\{:\}},\dots,0_{\{:\}}]$

$\mathbf{p}_l$



Sparsity: $|\mathbf{p}_l|$; Separation

6.MH-MCMC Algorithm

Fix $p_0=0\in\mathbb{R}^M$. For any $t\geq 0$, given $p_t\in\mathcal{P}$,

1. Generate a random variable Q_t with distribution $q(\cdot|p_t)$.
2. Generate a random variable

$$P_{t+1}=\begin{cases}Q_t, & \text{with probability } r(p_t,Q_t),\\ p_t, & \text{with probability } 1-r(p_t,Q_t),\end{cases}$$

where

$$r(p,q)=\min\left(\frac{v_q}{v_p},1\right).$$

3. Compute the least squares estimator $\hat{\theta}_{P_{t+1}}$.

4.Sparsity-Promoting Aggregation

Trade-off: MSE and sparsity

$$\hat{\boldsymbol{\lambda}}^{EW}=\arg\min_{\lambda\in\Lambda^L}\left(\sum_{l=1}^L\lambda_l\hat{r}_l+\frac{\beta}{M}\sum_{k=1}^L\lambda_k\log\frac{\lambda_k}{\pi_k}\right)$$

LS error: $\hat{r}_l=\dfrac{1}{M}\|\mathbf{y}-\mathbf{A}\hat{\mathbf{z}}_l\|_2^2$

Sparsity prior: $\pi_l=\dfrac{1}{H}\left(\dfrac{|\mathbf{p}_l|}{2eL}\right)^{|\mathbf{p}_l|}$

Consistency with L0-norm

$$\min_{\mathbf{p}\in\mathbb{C}^N}\|y-\mathbf{A}\boldsymbol{x}\|_2^2+\lambda\|\boldsymbol{x}\|_0$$

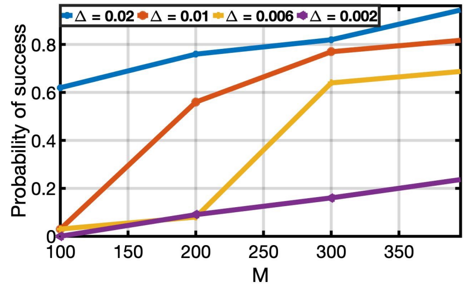
$\|\boldsymbol{x}\|_0\sim|\mathbf{p}_l|$

$$\min_{\lambda\in\Lambda^L}\sum_{l=1}^L\lambda_l\hat{r}_l+\lambda_l|\mathbf{p}_l|\log\frac{2eL}{|\mathbf{p}_l|}$$

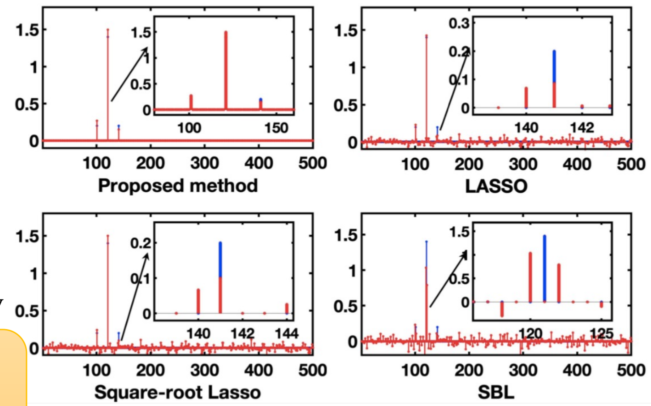
+

Structural constraint: separation

7.Simulation



Probability of Exact Support Recovery



Sample Recovered Signal Comparison

Observation:
Separation prior can improve the estimation accuracy