On Transmitter Polarization Optimization for Target Scattering Matrix Estimation

Kexiao Hu, Shuli Shi, Zhiwen Liu, Yougen Xu
School of Information and Electronics, Beijing Institute of Technology, Beijing, China
hukexiao1014@126.com, shishuli@bit.edu.cn, zwliu@bit.edu.cn, yougenxu@bit.edu.cn

SAM 2018

Abstract

- The knowledge about target scattering matrix is essential for adapted and matched polarimetric target illumination.
- The Cramer-Rao bound (CRB) for target scattering matrix estimation is known to be the function of transmitter polarization.
- To find the optimal transmitter polarization for target scattering matrix estimation, a suboptimal lattice search has to be used in the current method.
- We demonstrate that the mentioned CRB minimization for target scattering matrix estimation against compound Gaussian clutter can be achieved by simply transmitting several couples of probing pulses with orthogonal polarizations, for example, by transmitting alternately a number of horizontally and vertically polarized pulses just like the traditional scheme for directly measuring the target scattering matrix in the absence of clutter.

1. Introduction

- The target scattering matrix provides useful information of target. It is potentially useful for polarimetric detection of radar.
- The existing transmitter polarization optimization method aimed at CRB minimization in the context of target scattering matrix estimation with the assumption of compound Gaussian clutter is lattice searching method which is computationally unattractive.
- We present a simpler scheme for transmitter polarization optimization based on CRB minimization.

Major References

- Wang Jian et al. Adaptive polarimetry design for a target in compound-Gaussian clutter. SP, 2009, 89(6): 1061-1069.
- Wang Jian et al. Maximum likelihood estimation of compound-Gaussian clutter and target parameters. TSP, 2006, 54(10): 3884-3898.
- Wang Jian et al. Cramer-Rao bounds for compound-Gaussian clutter and target parameters. ICASSP, 2005, 1001-1004.

2. Measurement Model

Consider the simple coherent one-transceiver-antenna case and a point scatter in the far field. The target polarimetric scattering matrix is,

$$\mathbf{S}=\begin{bmatrix} s_{\mathrm{HH}} & s_{\mathrm{HV}} \\ s_{\mathrm{VH}} & s_{\mathrm{VV}} \end{bmatrix} \quad (1)$$

the received signal of the t-th echo pulse can be expressed as

$$\mathbf{r}(t)=\mathbf{S}\zeta(t)+\mathbf{e}(t) \quad (2)$$

where $\zeta(t)=[\zeta_{\mathrm{H}t},\zeta_{\mathrm{V}t}]^{\mathrm{T}}$ is the transmitted polarization vector, $\mathbf{e}(t)$ is the additive clutter vector of compound Gaussian distribution and can be written as

$$\mathbf{e}(t)=\sqrt{u(t)}\,\boldsymbol{\chi}(t) \quad (3)$$

After the vectorization transformation, the target scattering vector can be written as

$$\mathbf{x}=\begin{bmatrix} \mathrm{Re}[\mathrm{vec}(\mathbf{S}^{\mathrm{T}})] \\ \mathrm{jIm}[\mathrm{vec}(\mathbf{S}^{\mathrm{T}})] \end{bmatrix} \quad (4)$$

Then $\mathbf{r}(t)$ can be rewritten as

$$\mathbf{r}(t)=\varsigma(t)\mathbf{x}+\mathbf{e}(t) \quad (5)$$

Having a total of N echo pulses, we recast model (5) with a matrix form:

$$\bar{\mathbf{r}}=\mathbf{A}\mathbf{x}+\bar{\mathbf{e}} \quad (6)$$

where

$$\mathbf{A}=\begin{bmatrix} \zeta_{\mathrm{H1}} & 0 & \zeta_{\mathrm{V1}} & 0 & \zeta_{\mathrm{H1}} & 0 & \zeta_{\mathrm{V1}} & 0 \\ 0 & \zeta_{\mathrm{H1}} & 0 & \zeta_{\mathrm{V1}} & 0 & \zeta_{\mathrm{H1}} & 0 & \zeta_{\mathrm{V1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \zeta_{\mathrm{H}N} & 0 & \zeta_{\mathrm{V}N} & 0 & \zeta_{\mathrm{H}N} & 0 & \zeta_{\mathrm{V}N} & 0 \\ 0 & \zeta_{\mathrm{H}N} & 0 & \zeta_{\mathrm{V}N} & 0 & \zeta_{\mathrm{H}N} & 0 & \zeta_{\mathrm{V}N} \end{bmatrix}$$

$$\bar{\mathbf{r}}=[\mathbf{r}^{\mathrm{T}}(1)\ \ \mathbf{r}^{\mathrm{T}}(2)\ \ \cdots\ \ \mathbf{r}^{\mathrm{T}}(N)]^{\mathrm{T}} \quad (7)$$
$$\bar{\mathbf{e}}=[\mathbf{e}^{\mathrm{T}}(1)\ \ \mathbf{e}^{\mathrm{T}}(2)\ \ \cdots\ \ \mathbf{e}^{\mathrm{T}}(N)]^{\mathrm{T}}$$

3. Optimal Transmitter Polarization Design For Target Scattering Matrix Estimation

$u(t)$ is the texture component which is of inverse gamma distribution with mean one and shape parameter v.

The probability density function (PDF) of $u(t)$ is

$$p(u)=\frac{1}{\Gamma(v)}v^{v}u^{-v-1}\exp(-v/u) \quad (8)$$

$\mathbf{r}(t)\,|\,u(t)$ is of Gaussian distribution with mean $\varsigma(t)\mathbf{x}$ and covariance matrix $u(t)\boldsymbol{\Sigma}$. The conditional PDF of $\mathbf{r}(t)$ is

$$p_{r|u}(\mathbf{r}(t)\,|\,u(t)) =\frac{\exp\{-[\mathbf{r}(t)-\varsigma(t)\mathbf{x}]^{\mathrm{H}}[u(t)\boldsymbol{\Sigma}]^{-1}[\mathbf{r}(t)-\varsigma(t)\mathbf{x}]\}}{\det[\pi u(t)\boldsymbol{\Sigma}]} \quad (9)$$

The PDF of $\mathbf{r}(t)$ is

$$p(\mathbf{r}(t),u(t);\mathbf{x})=p_{r|u}(\mathbf{r}(t)\,|\,u(t))\,p_u(u(t)) \quad (9)$$

the marginal PDF of $\mathbf{r}(t)$ is：

$$p_{r|u}(\mathbf{r}(t)\,|\,u(t)) =\frac{\exp\{-[\mathbf{r}(t)-\varsigma(t)\mathbf{x}]^{\mathrm{H}}[u(t)\boldsymbol{\Sigma}]^{-1}[\mathbf{r}(t)-\varsigma(t)\mathbf{x}]\}}{\det[\pi u(t)\boldsymbol{\Sigma}]} \quad (10)$$

The CRBs for the target parameters are decoupled from the clutter parameters. Hence, the CRB matrix for target scattering coefficients remains same whether or not the clutter parameters are known. The transmitter polarization is adapted by minimizing the determinant of the CRB matrix. This is equivalent to maximizing the determinant of the Fisher information matrix (FIM).

The FIM entries are

$$[\mathbb{F}(\mathbf{x})]_{p,q} =\frac{2(v+2)}{v+3}\mathrm{Re}\left[\sum_{t=1}^{N}\frac{\partial\mathbf{x}^{\mathrm{H}}}{\partial\boldsymbol{\rho}(p)}\varsigma^{\mathrm{H}}(t)\boldsymbol{\Sigma}^{-1}\varsigma(t)\frac{\partial\mathbf{x}}{\partial\boldsymbol{\rho}(q)}\right] \quad (11)$$

where

$$\boldsymbol{\rho}=\left[\left[\mathrm{Re}[\mathrm{vec}(\mathbf{S}^{\mathrm{T}})]\right]^{\mathrm{T}},\left[\mathrm{Im}[\mathrm{vec}(\mathbf{S}^{\mathrm{T}})]\right]^{\mathrm{T}}\right]^{\mathrm{T}} \quad (12)$$

Let $\mathbf{C}$ be the transmitter polarization matrix,

$$\mathbf{C}=[\mathbf{c}_{\mathrm{H}}\ \ \mathbf{c}_{\mathrm{V}}]=\begin{bmatrix} \zeta_{\mathrm{H1}} & \zeta_{\mathrm{H2}} & \cdots & \zeta_{\mathrm{H}N} \\ \zeta_{\mathrm{V1}} & \zeta_{\mathrm{V2}} & \cdots & \zeta_{\mathrm{V}N} \end{bmatrix}^{\mathrm{T}} \quad (13)$$

the determinant of the FIM is

$$\det[\mathbb{F}(\mathbf{x})]=\left[\frac{2(v+2)}{v+3}\right]^{8}\det{}^{4}(\mathbf{C}^{\mathrm{H}}\mathbf{C})\det{}^{4}(\boldsymbol{\Sigma}^{-1}) \quad (14)$$

According to (13), for best target estimation of the scattering matrix, transmitter polarization optimization can be adopted to make $\mu_{\zeta_N}=\det(\mathbf{C}^{\mathrm{H}}\mathbf{C})$ as large as possible.

Let ,

$$\mathbf{B}=[\mathbf{b}_{\mathrm{H}}\ \ \mathbf{b}_{\mathrm{V}}]=\begin{bmatrix} \zeta_{\mathrm{H1}} & \zeta_{\mathrm{H2}} & \cdots & \zeta_{\mathrm{H}(N-1)} \\ \zeta_{\mathrm{V1}} & \zeta_{\mathrm{V2}} & \cdots & \zeta_{\mathrm{V}(N-1)} \end{bmatrix}^{\mathrm{T}} \quad (15)$$

Note that,

$$\mathbf{b}_{\mathrm{H}}^{\mathrm{H}}\mathbf{b}_{\mathrm{H}}+\mathbf{b}_{\mathrm{V}}^{\mathrm{H}}\mathbf{b}_{\mathrm{V}}=N-1 \quad (16)$$
$$\zeta_{\mathrm{H}N}^{*}\zeta_{\mathrm{H}N}+\zeta_{\mathrm{V}N}^{*}\zeta_{\mathrm{V}N}=\zeta_{N}^{\mathrm{T}}\zeta_{N}^{*}=1 \quad (17)$$

It follows that,

$$\mu_{\zeta_N}=\mathbf{b}_{\mathrm{H}}^{\mathrm{H}}\mathbf{b}_{\mathrm{H}}\mathbf{b}_{\mathrm{V}}^{\mathrm{H}}\mathbf{b}_{\mathrm{V}}-\mathbf{b}_{\mathrm{H}}^{\mathrm{H}}\mathbf{b}_{\mathrm{V}}\mathbf{b}_{\mathrm{V}}^{\mathrm{H}}\mathbf{b}_{\mathrm{H}} +(N-1)-\zeta_{N}^{\mathrm{T}}\mathbf{B}^{\mathrm{H}}\mathbf{B}\zeta_{N}^{*} \quad (18)$$

Note that maximizing is equivalent to minimizing the last term $\zeta_{N}^{\mathrm{T}}\mathbf{B}^{\mathrm{H}}\mathbf{B}\zeta_{N}^{*}$.

We conclude that the transmitter polarization adaptation via CRB minimization for the best target scattering matrix estimation under compound Gaussian clutter can be simply realized by submitting alternately multiple probing pulse pairs having orthogonal polarizations (AOTP).

4. Simulation Results

10 pilot signal are send firstly to guarantee the scattering matrix estimate efficiently, then three transmitter polarization modes are considered: 1) using fixed transmitter polarization; 2) using the optimal transmitter polarization proposed in [8]; and 3) using the proposed polarization optimization method, AOTP. The methods are compared in terms of the experimental mean square error (MSE) of scattering matrix estimate.

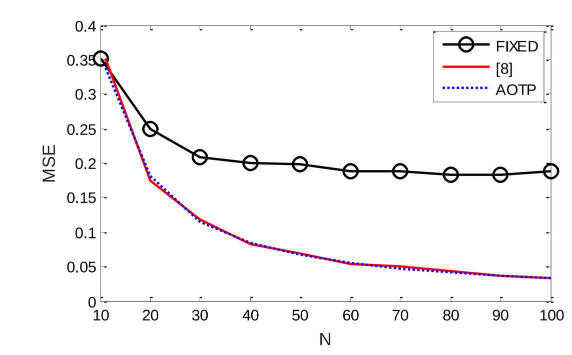

Fig. 1
MSE against N

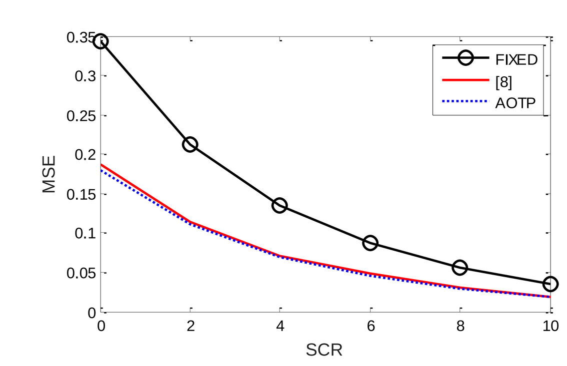

Fig. 2
MSE against SCR

we compare the average computation time of the two transmitter polarization modes under the following configurations:

1) CPU: Inter Core i7-6700 3.40 GHz;
2) Memory: 8.00GB;
3) System: 64bit Windows7;
4) MATLAB R2017a.

COMPUTATION TIMES COMPARASION

Method	the lattice searching method	the ATOP scheme
Times(s)	3531.869	0.092

5. Conclusions

We have demonstrated by both theoretical analyses and simulation study the requirement of only two different waveforms with orthogonal polarizations for CRB minimization-based transmitter polarization optimization in target scattering matrix estimation under compound Gaussian clutter. This leads to a simpler transmitter polarization optimization scheme compared to the current lattice searching technique.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (No. 61331019, No. 61490691).