Accuracy Evaluation Based on Simulation for Finite Precision Systems Using Inferential Statistics
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Context and Motivations

- Embed high complexity algorithms on over-constrained platforms
- Memory, energy, computation time constraints
- Approximate computing (ACx) trades-off quality and cost
- Data level ACx: Fixed-point arithmetic

Objectives

- Fixed-point refinement process
- NP-hard optimization problem
  \[ \min (C(w)) \quad \text{subject to} \quad P(w) < P_m \]
  - \( C(\cdot) \): implementation cost
  - \( w \): word-length vector
  - \( P(\cdot) \): noise power for word-lengths \( w \)
  - \( P_m \): noise power constraint
- Challenge: accuracy evaluation
- Noise power \( P(w) \) evaluation
- Simulation-based methods with a fixed number of samples \([10^6;10^{12}]\)

Quantization noise power characterization

- Quantization noise power \( P = E[e^2] \)
- Error distance \( e_i = |x_i - \hat{x}_i| \)
- \( x_i \): application output with fixed-point data types
- \( \hat{x}_i \): application output with floating-point data types
- Mean and standard deviation estimators
  \[ \mu_e^2 = \frac{1}{T} \sum_{t=1}^{T} e_i^2 \]
  \[ S^2 = \frac{1}{T} \sum_{t=1}^{T} (e_i^2 - \mu_e^2)^2 \]
- Inferential statistics
  - Confidence interval \( IC_{\mu_{e^2}} \)
    - Estimation of \( \mu_{e^2} \) included in \( IC_{\mu_{e^2}} \) with a probability \( p \)
    \[ IC_{\mu_{e^2}} = [\mu_{e^2} - a_{\mu_{e^2}}(p)\mu_{e^2} + a_{\mu_{e^2}}(p)] \]
    \[ a_{\mu_{e^2}}(p) = z_{\alpha}(p) \cdot \frac{S}{\sqrt{N_p - 1}} \]
  - Minimal number of samples to simulate for a desired accuracy \( h \)
    \[ N_p > \frac{\alpha^2}{h^2} \cdot S^2 \]

Algorithm to compute \( N_p \)

Algorithm 1 Proposed Computation of \( N_p \)

\[ E \leftarrow \emptyset \]
\[ n \leftarrow 0 \]
\[ \text{repeat} \]
\[ (e_{n+1}^2, \ldots, e_{n+T}^2) \leftarrow \text{sampling}(E, T) \]
\[ E \leftarrow E \cup (e_{n+1}^2, \ldots, e_{n+T}^2) \]
\[ \mu_{e^2} \leftarrow \text{computeMean}(E, n + T) \]
\[ S^2 \leftarrow \text{computeSD}(E, n + T; \mu_{e^2}) \]
\[ N_p \leftarrow \text{computeN}(S^2; h) \]
\[ n \leftarrow n + T \]
\[ \text{until } n \geq N_p \]
\[ \text{return } N_p \]
\[ \text{end procedure} \]

Exploitation for Word-Length Optimization

- Adaptation of the number of samples to simulate
- Comparison with a threshold (constant) \( c \)
  - Minimal number of samples to take a decision i.e. \( c \notin IC_{\mu_{e^2}} \)

Experiments: quality results

- \( N_p \) for varied elementary blocks and \((h, p)\).

<table>
<thead>
<tr>
<th>( h )</th>
<th>( 0.001 )</th>
<th>( 0.0001 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_m )</td>
<td>( 10^6 )</td>
<td>( 10^{12} )</td>
</tr>
<tr>
<td>( C )</td>
<td>( 65 )</td>
<td>( 68.4 )</td>
</tr>
<tr>
<td>( C_{\text{Sigma}} )</td>
<td>( 95 )</td>
<td>( 95 )</td>
</tr>
<tr>
<td>( N_p )</td>
<td>( 98 )</td>
<td>( 98.7 )</td>
</tr>
<tr>
<td>( N_{\text{Sigma}} )</td>
<td>( 95 )</td>
<td>( 95 )</td>
</tr>
<tr>
<td>( \text{Quantization} )</td>
<td>( 95 )</td>
<td>( 95 )</td>
</tr>
<tr>
<td>( 8\text{-bit to 6\text{-bit}} )</td>
<td>( 91 )</td>
<td>( 91 )</td>
</tr>
<tr>
<td>( 8\text{-bit} )</td>
<td>( 95 )</td>
<td>( 95 )</td>
</tr>
<tr>
<td>( \text{Power Constraint} )</td>
<td>( 91 )</td>
<td>( 91 )</td>
</tr>
</tbody>
</table>

- Exploitation to accelerate Word-Length Optimization (WLO).
  - Reference WLO: simulation on \( N_{\text{Ref}} = 100000 \) samples
  - \( G_{\text{Ref}} \): gain in terms of number of simulated samples
  - \( G_{\text{Ref}} \): gain in terms of simulation time
  - \( \alpha \): average ratio between the effective number of simulated samples and the minimal number of samples \( N = 30 \).

<table>
<thead>
<tr>
<th>Applications</th>
<th>Cost</th>
<th>( C_{\text{Ref}} )</th>
<th>( G_{\text{Ref}} )</th>
<th>( \alpha )</th>
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<tbody>
<tr>
<td>FIR</td>
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<td>824</td>
<td>769</td>
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<tr>
<td>Stereo</td>
<td>16</td>
<td>1.01</td>
<td>972</td>
<td>893</td>
</tr>
</tbody>
</table>

**Note:** The table above represents the accuracy evaluation results for different applications using WLO methods. The columns include the type of application, cost, reference power constraint, gain, and average ratio. The rows detail the specific values for each application.