Atomic Norm Minimization Based Range-Direction Indication For Frequency Diverse Array: A Matrix Completion Perspective

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**Contribution**

- **Purpose:** Achieve range-direction indication in Frequency Diverse Array (FDA) without coupling and high sidelobe.
- **Key idea:**
  - Regard the FDA as a 2-D sampling on the spatial-frequency domain;
  - Use Atomic Norm Minimization (ANM) to complete the missing observation.
- **Performance:**
  - Achieve a 2-D sinc-like structure beampattern;
  - Indicate targets successfully and accurately.

**Method & Framework**

1. **Sampling on spatial-frequency domain**
   - We regard FDA as the joint sampling of the targets’ range-direction information on the 2-D spatial-frequency domain.
   - FDA receives data $\mathbf{y}$, the orthogonal projection operator onto the subspace of vector supported on $\mathcal{T}$. Define $\mathcal{T}$ on the frequency diverse code $m_n$ as
     \[
     \mathcal{T} = \{(n-1)M + m_n|n = 1,2,\ldots,N\}
     \]
   - All the existing FDA methods can be regarded as sampling on the 2-D spatial-frequency domain just with different sampling type.
   - $\mathbf{x}$: the full observation matrix which can be formulated as each antenna transmits all the $M$ monostatic carriers at the frequency ranges from $f_0$ to $f_0 + (M-1)\Delta f$, and afterwards receives all the reflected echoes with an orthogonal waveform design,
     \[
     \mathbf{x} = \sum_{i=1}^{K} \alpha_i(f_i)\alpha_i(f_0)^T
     \]
   - where $\alpha_i(f_i) = \left[1, e^{-j2\pi f_i r_1}, e^{-j2\pi f_i r_2}, \ldots, e^{-j2\pi f_i r_M} \right]^T$ and $\alpha_i(f_0) = \left[1, e^{-j2\pi f_0 r_1}, e^{-j2\pi f_0 r_2}, \ldots, e^{-j2\pi f_0 r_M} \right]^T$

2. **Matrix Completion by Atomic Norm Minimization**
   - Adopt Atomic Norm Minimization (ANM) to reconstruct the full matrix $\mathbf{X}$. The corresponding atomic set is defined as the collection of all the 2-D complex sinusoids:
     \[
     \mathcal{A} \triangleq \{ \alpha(f_0,f_i) = \alpha(f_0)\alpha(f_i)|f_0 \in (0,1), f_i \in (0,1)\}
     \]
   - The atomic norm is defined as
     \[
     \|x\|_A \triangleq \inf \left\{ \sum_{i=1}^{K} \alpha_i(f_i)\alpha_i(f_0)^T | x = \sum_{i=1}^{K} d_i \alpha(f_i)\alpha(f_0)^T \right\}
     \]
   - So the reconstruction of the full matrix $\mathbf{X}$ can be arranged as the following optimization problem
     \[
     \hat{x} = \arg\min \|x\|_A \quad s.t. \quad \mathbf{P}_f(x) = y
     \]
   - where $x = \text{vec}(\mathbf{X})$. This problem can be solved by a semi-definite programming (SDP) algorithm.

3. **Range-Direction Indication Using the Full Data**
   - Then we use the full observation vector $\mathbf{x}$ to obtain the range and direction of the targets. For the single-snapshot case (termed the Single Measurement Vector (SMV) scenario), the maximum likelihood estimation can be obtained by a replica-correlation, so the spatial-frequency spectrum can be estimated as
     \[
     \mathbf{P}(\mathbf{r}, \theta) = c(f_0, f_i)^H \mathbf{x}
     \]
   - If there are $L > 1$ snapshots, we can form all the full vectors $\mathbf{x}(t)$ as a Multiple Measurement Vector (MMV) scenario. Then we can use some methods such as MUSIC and Capon to achieve a super resolution.

**Simulation Results**

FDA consists of $N = 16$ antennas, $f_0 = 96$ Hz, $\Delta f = 200$ kHz. Two targets: $r_1 = 10\, \text{km}$, $\theta_1 = 30^\circ$ and $r_2 = 10\, \text{km} + 75\, \text{m}$, $\theta_2 = 45^\circ$. The power level of Target 1 is $5\, \text{dB}$ larger than that of Target 2.

- **2-D sinc-like beampattern; Accurately indication of targets.**

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