**Motivation**

- The smart grid will be composed of smaller grids, known as MicroGrids (MGs), in which energy is locally generated and consumed.
- MGs can operate in islanded mode, i.e., without connection to the main grid.
- MGs improve reliability in power delivery and efficiency of energy usage.

- Demand Response (DR) programs are another alternative for reducing costs at the MG.
  - Low energy consumers have been traditionally ignored in DR programs due to their reduced impact.
  - DR Aggregators (AGs) have recently appeared in new market agents, capable of controlling and managing groups of small energy consumers, granting omnipresent access to DR programs.

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**Energy trading without demand response**

\[ \text{Problem formulation:} \]

Each MG designs its energy generation and trading strategy, \( y_m = [g_m, a_m] \), to maximize its own benefit, i.e., the difference between incomes and costs (generation + energy transfer costs):

\[ \min \sum_{m \in \mathcal{M}} \left( \sum_{t \in \mathcal{T}} (1 - \varepsilon_{\text{ref,}m,t}) y_m, t \right) \]

**Design constraints:**

1. The energy generated by the \( m \)-th MG must satisfy \( \sum_{t \in \mathcal{T}} y_m, t \leq \sum_{t \in \mathcal{T}} g_m, t \) for all \( t \in \mathcal{T} \).
2. The total energy must satisfy \( \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} y_m, t \leq \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} g_m, t \) for all \( t \in \mathcal{T} \).
3. Energy storage limits are given by \( E_{\text{in,}m} \) and \( E_{\text{out,}m} \) for all \( m \in \mathcal{M} \).
4. Load balancing: \( \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} y_m, t = 0 \) for all \( t \in \mathcal{T} \).

**1-player Generalized Nash Equilibrium Problem (GNEP):**

The \( m \)-th MG revenue maximization problem can be written as follows:

\[ \min_{x_m \in \mathcal{E}(y_m)} f_m(y_m), \]

where the feasible set of MG is \( y_m \), and is coupled with the strategy of the other MGs, \( a_m \in \mathcal{A}(y_m) \).

**Variational solutions to the GNEP:**

**Proposition 1.** The variational solutions of the GNEP defined by (2), \( y_m \in \mathcal{M}, 1, \ldots, M \), are solutions of the following Network Utility Maximization (NUM) problem:

\[ \min_{\tilde{y}_m} \sum_{m \in \mathcal{M}} \tilde{y}_m, \]

where \( y_m \) contains the strategy of the different MGs \( y_m \) and \( \tilde{y}_m \) with \( \tilde{y}_m = |g_m| y_m \) and \( a_m \) and its associated feasible set is \( \tilde{y}_m \). Additionally, the converse implication holds true as well.

**Simulation Setup**

- \( M = 2 \) MGs and one AG.
- \( \gamma_n = 2008/1000 \) MW.
- \( \gamma_n = 0.12 \) MW. Oil generators: \( G_1 = [0, 0.12] \) with \( \gamma_n = 2008/1000 \) MW.
- Oil generators: \( G_2 = [0, 0.12] \) with \( \gamma_n = 2008/1000 \) MW.

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**Energy trading with DR aggregators**

**Problem formulation:**

- MGs can shift trade loads from \( x_u \) to \( x_u \) to reduce MGs costs.
- \( m \)-th AG offers a fraction \( \Gamma_m \in [0, 1] \) of its savings to the MGs. In particular, AG \( \alpha \) receives \( \Gamma_m \delta f_m(y_m) \).

**Design constraints:**

- Design constraints 1, 2, and 3, 1 in the left column.
- Load balancing: \( \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} y_m, t = 0 \) for all \( t \in \mathcal{T} \).
- All requested loads must be scheduled within the time horizon \( t \), i.e.,

**Simulations**

- Obtain the variational solutions of the GNEP by solving (3) in a distributed way by means of dual decomposition.