This paper proposes an extended conjugate polar Fourier transform (ECPFT), to design iterated radial filter bank (RFB) and directional filter bank (DFB) convenient for accurate multiscale and multidirectional decomposition in discretization over a convolution network. With conjugated symmetric form, ECPFT would convert complex directional wavelets in original spatial domain to real ones in the inverse Fourier domain of ECPFT. Furthermore, it can contribute to changing the design of nonseparable RFB and DFB to decomposition in scales and directions with 1-D filter bank in each dimension separately in the ECPFT domain. The generated properties of conjugate symmetry and periodicity of 2π in both angular and radial dimensions, also guarantee convolution and sampling operations in both dimensions within the inverse Fourier domain of ECPFT. The fast algorithm for discretization is developed to reduce complexity.

**Abstract**

**Experiments**

<table>
<thead>
<tr>
<th>Fast approximation</th>
<th>RFB &amp; DFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>size</td>
<td>128</td>
</tr>
<tr>
<td>128</td>
<td>256</td>
</tr>
<tr>
<td>128</td>
<td>512</td>
</tr>
<tr>
<td>512</td>
<td>1024</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>energy ratio</th>
<th>PSNR</th>
<th>memory</th>
<th>runtime (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.66%</td>
<td>21.9</td>
<td>0.21%</td>
<td>0.89%</td>
</tr>
<tr>
<td>0.66%</td>
<td>21.9</td>
<td>0.21%</td>
<td>0.89%</td>
</tr>
<tr>
<td>0.21%</td>
<td>0.89%</td>
<td>0.21%</td>
<td>0.89%</td>
</tr>
</tbody>
</table>

Since the input signal is divided into two parts x₁ and x₂, x₁ is first recovered from decomposition coefficients and then combined with x₂ to obtain the reconstructed signal. In discrete case, different orders have different effects.

**Table 1.** Energy ratio and Running time of calculating Xₖₐᵢ for both direct formula and fast approximation.

**Table 2.** PSNR of reconstructed images with different transforms.

\[
\begin{array}{cccc}
\text{PSNR(dB)} & \text{Methods} & \text{Wavelet} & \text{Curvilinear} & \text{Proposed} \\
\hline
\text{barbara} & 17.68 & 22.28 & 21.97 & 22.86 \\
\text{cameraman} & 17.33 & 25.33 & 23.48 & 26.70 \\
\text{peppers} & 17.93 & 25.04 & 22.0 & 25.41 \\
\text{fingerprint} & 17.56 & 19.9 & 22.95 & 23.01 \\
\text{lenna} & 16.97 & 20.87 & 20.48 & 26.70 \\
\text{baboon} & 16.87 & 23.59 & 24.95 & 25.41 \\
\text{bridge} & 16.00 & 21.75 & 20.62 & 22.01 \\
\text{lenna} & 19.16 & 21.75 & 21.50 & 26.70 \\
\text{woman} & 19.43 & 25.04 & 24.17 & 25.95 \\
\text{crowd} & 19.68 & 23.08 & 22.22 & 23.92 \\
\text{mildrop} & 22.26 & 27.41 & 27.37 & 27.75 \\
\text{plane} & 18.92 & 22.78 & 23.46 & 23.72 \\
\end{array}
\]

**Nonlinear Approximation**

For the 12 test images, we select 3% most significant coefficients in each transform domain, and then compare the reconstructed images from these sets of coefficients.